A STOCHASTIC EVALUATION OF THE SHORT-RANGE ECONOMIC ASSUMPTIONS IN THE 1994 OASDI TRUSTEES REPORT

by
Richard S. Foster

Actuarial Study No. 109

U.S. Department of Health and Human Services
Social Security Administration
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Additional copies may be obtained from:

Social Security Administration
Office of the Actuary
Room 700, Altmeyer Building
Baltimore, MD 21235

Voice: (410) 965-3015
Fax: (410) 965-6693

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Forword

Actuarial estimates of future risk-based financial operations are necessarily uncertain, whether they are for life insurance, pensions, social insurance programs, or other income-security arrangements. Many efforts have been made over the years to better understand the degree of uncertainty present in such estimates. This actuarial study represents the first inquiry by the Office of the Actuary into using statistical time series analysis as a tool to help quantify the uncertainty associated with the actuarial projections developed for the Social Security program. This first step is somewhat tentative, in recognition of our relative lack of experience in this area. It is hoped that others will confirm these results and improve upon them, and use this study as a springboard for further research using time series techniques.

Based on the results of this study, I believe that statistical time series analysis has the potential to improve our understanding of the possible range of variation that can reasonably be expected for future trust fund operations. Ultimately, however, application of these techniques will not provide the perfect crystal ball—or even a mediocre one. The future will remain unknowable and we will continue to be surprised by unusual economic developments. Nonetheless, if the effort helps sharpen our assessment, even slightly, of Social Security's ability to fulfill its income-security obligations, then it will be time well spent.

I would like to thank a number of individuals who provided assistance with this effort. In the Office of the Actuary, Steve McKay, Eli Donkar, and Steve Goss reviewed the draft report and provided many insightful comments, and Rob Baldwin assisted with data development. Special thanks are due Jeff Kunkel, who assisted with all aspects of the report's preparation with extraordinary diligence and patience. Ken Sander and Pat Skirvin of the Office of Research and Statistics assisted with the data gathering and provided much helpful advice and constructive criticism. I especially wish to acknowledge the invaluable comments and suggestions provided by Dr. Nathan S. Balke of the Department of Economics, Southern Methodist University, and Dr. Jonathan D. Cryer of the Department of Statistics, University of Iowa. Also, Dr. Kenneth F. Kroner of the Department of Economics, University of Arizona, generously made available his Fortran routines for estimating GARCH time series models. Last but not least, Sheldon Baker of the Office of the Actuary painstakingly reviewed the final report for typographical errors and inconsistencies.

Any opinions stated in this report are solely my own and no official support or endorsement by the Social Security Administration is intended or should be inferred.

Richard S. Foster
Deputy Chief Actuary

August 1, 1994

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Summary

This study evaluates four of the key short-range economic assumptions underlying the 1994 annual report of the Board of Trustees of the Social Security trust funds. The evaluation focuses on the likelihood that actual future values of the economic variables will fall within the range defined by the Trustees' "low-cost" and "high-cost" assumption alternatives. In addition, the relative likelihood of the "intermediate" assumptions is considered.

The evaluation is performed using statistical time series techniques, following the recommendations of the Technical Panel of actuaries and economists convened by the 1991 Advisory Council on Social Security. A statistical time series, or *stochastic*, model is estimated for each of the economic variables and used to produce forecasts and forecast probability intervals for the period 1994 through 1998. The Trustees Report assumptions are evaluated in comparison to these forecasts and their measures of uncertainty.

For three of the four variables studied—the inflation rate, unemployment rate, and real wage increase—the estimated probability that the average future value of the variable will fall within the range defined by the Trustees Report alternatives is roughly 50 percent. The corresponding probability for the real interest rate is significantly lower, at 26 percent. For consistency, consideration should be given to the possibility of broadening the assumption range for the real interest rate. Whether the range encompassed by the Trustees Report assumptions should be wider or narrower than 50 percent is an issue that cannot be evaluated based on *univariate* (individual) analysis of the economic time series, as used in this report. Future research, based on a *multivariate* examination of the data, may help answer this question.

The intermediate inflation rate and real interest rate assumptions are found to be very close to the "most likely" forecasts from the stochastic models. For the unemployment rate and the real wage increase, however, the estimated probability of a result more favorable than alternative II is substantially lower than the probability of a more adverse result. In each instance, the "most likely" forecast from the stochastic model coincides approximately with the "high-cost" or "pessimistic" assumptions of the Trustees Report. These results warrant further review; if no fault or bias can be found with respect to the underlying stochastic models, then careful consideration should be given to the possibility that the unemployment and real wage assumptions are too optimistic in the short range.

The validity of these results depends critically on whether appropriate stochastic models have been identified and accurately estimated, and on the implicit assumption that the future will continue to behave similarly to the past. The results of this study suggest that statistical time series analysis offers a potentially valuable addition to the traditional tools used in developing economic assumptions for the Trustees Report

I. Introduction

The financial outlook for the Social Security trust funds depends upon the balance between future income and expenditures. These in turn will be determined by the number of workers paying taxes in the future and their average taxable earnings, together with the number of persons receiving benefits and their average benefit amounts. Projecting these elements necessitates the use of numerous assumptions as to future economic conditions and demographic trends.

As a result of the uncertainty inherent in such undertakings, the Board of Trustees for the Social Security trust funds makes use of three alternative sets of economic and demographic assumptions. An "intermediate" set of assumptions is selected to represent the most likely outlook or "best estimate." "Low-cost" and "high-cost" alternatives are also shown to indicate the effect on the program's financial status of more favorable or less favorable economic and demographic conditions, respectively. Together, the three sets of assumptions provide an illustration of the range and possible variation in future financial operations for the trust funds. The results of the Trustees' financial evaluation of the Social Security program are presented in their annual report to Congress.[1]

The Trustees Report assumptions are developed through the collective analysis, judgment, and negotiation of actuaries, economists, and other staff at the Social Security Administration and the Departments of Treasury, Labor, and Health and Human Services. In 1990, the Trustees Report assumptions were reviewed by an independent team of actuaries and economists convened by the Quadrennial Advisory Council on Social Security.[2] While their final report stated, "Generally, the Panel found the Agency's projection work to be professional and highly competent," they nonetheless made a number of recommendations concerning the assumptions and the methodology used in setting the assumptions.

The Panel was particularly interested in the "development of methods to quantify the uncertainty of short- and long-range forecasts, both for particular assumptions and [the resulting] projections." They wrote:

> Further work is necessary to define the conceptual framework for the current low- and high-cost projections. Although theoretically they represent a collection of extreme values for each of the variables, how they should be interpreted is not obvious. For example, possible interpretations include: absolute bounds on what could possibly happen, confidence intervals, illustrative alternative projections, and sensitivity analyses.

The technical experts also called for specific further research in several areas, including the following:

> [A] major conclusion that emerges from this review is that empirical research could assist in resolving the major issue debated by the Panel. Specifically, current Social Security long-range forecasting procedures rely heavily on historical averages of relevant variables taken over different horizons. Disagreements center on which averaging period or which averaging weights are appropriate for each series. These questions can be addressed by using the statistical techniques of time series analysis. Resources should be devoted to the development of alternative statistical approaches for

[1] See reference [19].

[2] See reference [11].

extrapolating past time series of relevant variables. Initially, the focus should be on univariate approaches, though the Panel suspects that multivariate approaches might ultimately be worthwhile.

This actuarial study presents the initial results of an effort to implement these recommendations. In particular, univariate statistical time series techniques have been used to evaluate the key economic assumptions in the 1994 Trustees Report for the period 1994-98. The primary focus is on the range of variation encompassed by the low-cost and high-cost sets of assumptions in the short range. To this end, the patterns of past variation in the economic time series were analyzed and used to forecast the likely range of future variation. This range was then compared to the low-cost and high-cost sets of assumptions. In addition, the intermediate Trustees Report assumptions were examined to determine where they fell within the range of future variation suggested by the statistical models.

Four economic time series were studied: The rate of inflation (as measured by the Consumer Price Index), the unemployment rate, the real interest rate, and the increase in real average wages.[3] Collectively, these factors have the greatest influence on the financial operations of the Old-Age and Survivors Insurance (OASI) and Disability Insurance (DI) Trust Funds in the short range. Quarterly data from approximately 1950 through 1993 formed the basis for the study (with the exception of real interest rates, where data based on current trust fund investment policies were available for 1961 and later).

The Trustees Report economic assumptions were evaluated in comparison to the forecasts produced by an appropriate statistical time series, or *stochastic*, model of the *autoregressive integrated moving average* (ARIMA) variety.[4] Such models have been widely used in the evaluation and forecasting of economic (and other) time series for many years. Where necessary, the models were modified to accommodate unusual statistical characteristics—such as outliers, level shifts, or non-constant variance—using the relevant time series techniques developed for these purposes by researchers in recent years.

By definition, ARIMA models take the form

$$y_t = \phi_0 + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q}$$

where y_t is the value of the economic variable at time t, and ε_t is the difference between the actual value of the variable and the model estimate at time t (referred to as the *error term*). The coefficients $\phi_1, \dots, \phi_p$ and $\theta_1, \dots, \theta_q$ are the *parameters* of the model and are estimated in such a way as to minimize the sum of the squared errors ε_t or to maximize the *statistical likelihood* of the particular model.

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[3] The term "real" is used to indicate the net level of a variable after adjustment for inflation. For example, a 6-percent interest rate, after adjustment for inflation of 4 percent, results in a real interest rate of about 2 percent.

[4] A series of data values that follows an underlying set of probability rules over time is said to be generated by a *stochastic process*. Textbooks by Box and Jenkins [1], Cryer [3], Miller and Wichern [6], Mills [7], and Pankratz [8] all provide excellent introductions to time series analysis using ARIMA models.

Thus, these models relate the current value for a variable to the variable's past values in the preceding p periods and to the model's errors for the past q periods. In this context, "periods" may be years, quarters, months, weeks, or even days. In this study, all of the stochastic models were estimated using quarterly data. This technique can be contrasted to the regression techniques employed in traditional econometric analysis, where a "dependent" variable is estimated based on the values of certain "independent" or "explanatory" variables. For example, wage increases might be estimated from data on price increases (inflation), productivity gains, and labor market conditions. In contrast, ARIMA time series models would generally estimate wage increases solely on the basis of past wage increases. Discussions of the advantages and disadvantages of each approach can be quite entertaining[5] but exceed the scope of this report.

Once an appropriate stochastic model was estimated for an economic variable, it was used to generate prediction intervals for the 20-quarter period 1994:I through 1998:IV. These prediction intervals represent the range within which the economic variable is expected to fall, for a specified level of "certainty." For example, based on data through 1993:IV, the model for the unemployment rate predicts that there is a 90-percent likelihood that the actual unemployment rate in 1994:I will fall within the range of 5.6 to 6.8 percent. If the statistical model is properly specified and estimated, and if the future continues to behave like the past, then we would expect the actual value to fall within the prediction interval about 90 percent of the time—and to fall outside the interval about 10 percent of the time.[6]

The conditions mentioned briefly in the prior paragraph deserve careful attention. While stochastic models can produce exact-sounding forecasts and forecast intervals, to any degree of precision desired, ultimately there can be no assurance that "real life" will behave like the forecasts. In particular, the process is subject to the following three major caveats:

1) Real life is infinitely complicated. The stochastic models employed in this analysis rely on very simple relationships and do not attempt to model the true set of underlying conditions, interactions, motivations, external influences, etc. that affect economic conditions. Moreover, the patterns of variation experienced in the past may not continue in the future; new, unanticipated events may occur and have significant consequences for economic growth and stability. In short, the U.S. economy may not be sufficiently impressed with our research findings to adapt its behavior to them.

2) Even if the economic variables under study can be suitably approximated by a statistical time series model, we must identify the proper form or specification for the model. Identification of the proper model is not an exact science, nor is there necessarily a unique best answer.

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[5] See, for example, section 9.4 of Granger and Newbold [5].

[6] The actual unemployment rate in the first quarter of 1994 subsequently turned out to be about 6.1 percent (estimated on a basis consistent with the Bureau of Labor Statistics' "old" definition of this variable). In this instance, the actual figure was well within the 90-percent prediction interval estimated by the statistical model.

3) For a given specification, the parameters of the model must be estimated based on the available data. The data may be subject to measurement errors or inconsistent definition through time, and may be available only for a relatively short period. Thus, the parameter estimates will normally differ somewhat from the "true" values, with a corresponding effect on the accuracy of any forecasts based on the model.

For these reasons, stochastic models are at best a tool that can facilitate analysis of the patterns of variation that occur in time series and provide helpful, if limited, information about the characteristics of the variables under study. The future may well be very different from the past, in ways that we cannot hope to anticipate or incorporate into models. This concern, of course, applies to *all* methods of forecasting and is not unique to statistical time series analysis. Given these constraints and limitations, stochastic models can help address the basic issue at hand: how to develop reasonable and plausible forecasts of particular variables of interest, based on the available information to date. Just as the process of selecting assumptions for the Trustees Report ignores the possibility of Armageddon in the year 2000, so too do the stochastic models reflect a "normal" outlook, without consideration of extraordinary, unexpected developments.

In this study, the quarterly assumptions for the next 5 years from the 1994 Trustees Report are compared to a corresponding set of forecast intervals from the associated stochastic model. Comparisons are made both on a quarter-by-quarter basis, reflecting the specific assumptions in the Trustees Report, as well as on the basis of the *average* level assumed for each variable over the 5-year period. Both approaches provide useful comparisons and help establish quantitative measures for the range of variation between the low-cost and high-cost assumptions in the Trustees Report.

The results of these comparisons for the inflation rate, unemployment rate, real interest rate, and real wage increase are shown in the following sections II through V of the report, respectively. A concluding summary of the results and suggestions for further research are provided in section VI. Appendix A presents a detailed summary of the methodology used to identify, estimate, and test the statistical models. The models estimated for each of the four economic time series are described in appendices B through E, respectively. Appendix F lists the historical data for the four economic time series.

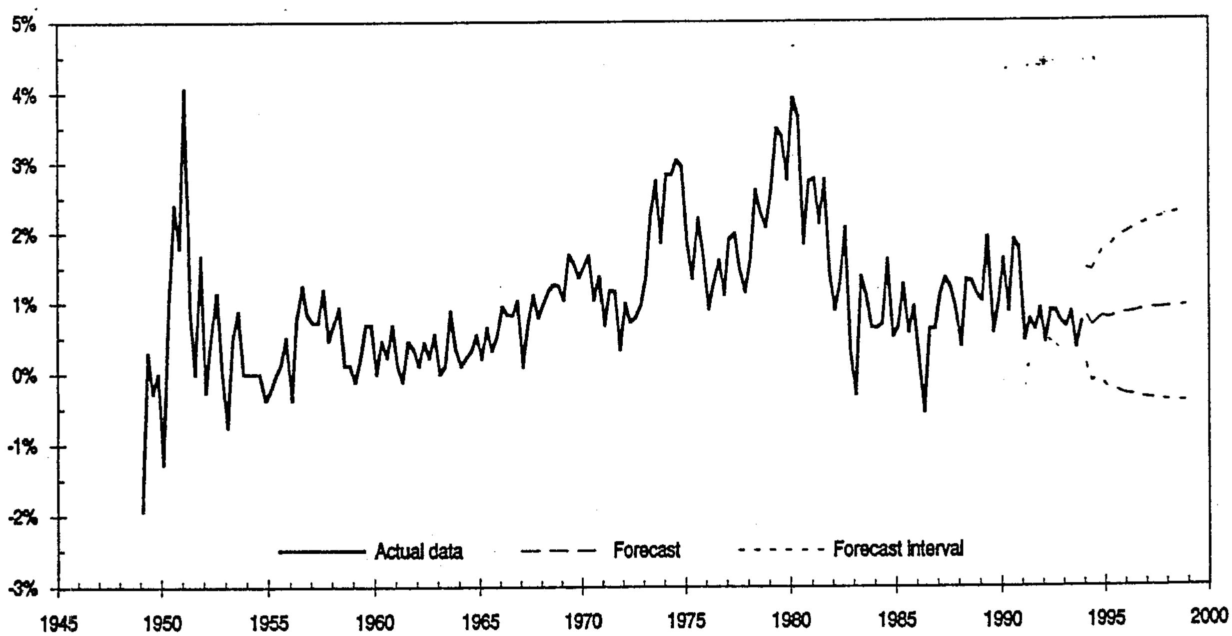
Finally, it is important to remember that the research findings presented in this study represent only the preliminary steps in the process envisioned by the Advisory Council's panel of actuaries and economists. The stochastic models developed here can be used to help describe the range of variation encompassed by the Trustees Report assumptions. The models offer little guidance, however, concerning whether that range is too narrow, too wide, or about right when the economic assumptions are considered collectively rather than individually. *Multivariate* statistical time series techniques can help to address these concerns and should be the focus of future research.

II. Inflation Rate

The rate of inflation is an important determinant of Social Security expenditures in the short range because benefit payments are directly tied to inflation through the annual cost-of-living adjustment (COLA). A period of rapid inflation, with prices increasing more rapidly than wages, can have a serious, adverse effect on the financial status of the Social Security program. Such conditions contributed directly to the OASDI financing crises experienced in the mid-1970s and again in the early 1980s. In addition to its near-term effects, inflation also influences the long-range outlook for the Social Security program, as indicated in the sensitivity analysis shown in table II.G5 of the 1994 Trustees Report.

Chart 1 shows the quarterly percentage increase in the CPI starting with the first quarter of 1949 and extending through the last quarter of 1993.[7] The inflation rate showed substantial variation in the early 1950s, settled down to relatively stable and moderate levels from the mid-1950s through the mid-1960s, and then began to increase significantly in the late 1960s. Prices surged in the mid-1970s and again in the late 1970s, in part as a result of oil shortages, and then returned to much lower levels following the major recession in the early 1980s.

Chart 1—Quarterly inflation rate for 1949-93 and forecast for 1994-98 with 90-percent forecast interval



[7] The data shown are based on the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W), without seasonal adjustment. This series is used to determine the annual cost-of-living adjustment for OASDI benefits. The inflation rate is calculated as the increase in the CPI-W from one quarter to the next and is not adjusted to an annual basis.

As indicated in table 1, the quarterly inflation rate averaged 0.99 percent during 1949-93 (equivalent to an annual rate of 4.02 percent) and had a standard deviation of 0.94 percent. Quarterly price increases ranged from a high of 4.05 percent (1951:I) to a low of –1.93 percent (1949:I). The 50th percentile or median of the inflation rate data was 0.87 percent.

Table 1—Summary statistics for quarterly inflation rate

Period	Number of points	Mean	Standard deviation	Minimum value	25th percentile	50th percentile	75th percentile	Maximum value
1949-93...	180	0.99%	0.94%	–1.93%	0.39%	0.87%	1.37%	4.05%

The appearance of the data in chart 1 suggests that inflation rates from one quarter to the next are not independent. In other words, high rates of inflation tend to be followed by additional high rates, and low rates tend to be followed by additional low rates. This characteristic is typical of many economic time series and is well-suited to representation by ARIMA statistical models.

The time series model established for the inflation rate data is shown below. A detailed discussion of its derivation and characteristics is available in appendix B.

$$c_t = 0.1342 + 0.5409c_{t-1} - 0.0010c_{t-2} + 0.3724c_{t-3} - 0.0474c_{t-4} + \varepsilon_t$$

This formula estimates the inflation rate at time t, designated c_t, as a weighted average of the corresponding rates in the prior 4 quarters (c_{t-1}, c_{t-2}, c_{t-3}, c_{t-4}). The factor ε_t represents a random "shock" or "error" affecting the inflation rate each quarter.

In standard ARIMA models, the error term is assumed to be normally distributed with a mean of zero and a constant standard deviation. The inflation rate data, however, is not consistent with the assumption of a constant standard deviation. Consequently, the stochastic model is modified in a way that permits the standard deviation of the error term to change through time. This modification allows the model to better reflect the alternating periods of calm and volatility shown by the inflation data. Implementation of the modification involves a second equation, for estimating the *variance* of the error term (the standard deviation squared).[8] The variance at time t, designated as σ_t^2, is modeled as a function of the variance in the prior quarter, σ_{t-1}^2, and the error term in the prior quarter ε_{t-1} as shown below.

$$\sigma_t^2 = 0.0250 + 0.1678\varepsilon_{t-1}^2 + 0.7441\sigma_{t-1}^2$$

A formal search was performed for statistical *outliers* and *level shifts* in the inflation rate data. The existence of such unusual occurrences can affect the identification and estimation of time series models and, in some instances, may affect forecast accuracy and/or the uncertainty associated with the forecast. To avoid such problems, it is sometimes necessary to incorporate the

[8] In practice, the parameters of both equations are estimated jointly, in such a way as to maximize the statistical likelihood of the model. The equations shown above for the inflation rate and its error variance are the results of this process.

outliers and level shifts into the model. Doing so, however, may understate the expected future range of variation (to the extent that such events may continue to occur from time to time in the future). In this report, outliers and level shifts were incorporated into the stochastic model only when their omission would have significantly biased the specification or the estimation of the model, or if the level of the model's central forecast would be affected. In all other cases, outlying events and shifts were not incorporated, to avoid understating the likely future range of variation. Appendix A provides additional information on this topic.

For the inflation rate data, the search for outliers and level shifts identified a number of each but none was incorporated into the stochastic model. Their exclusion did not significantly affect the specification or estimation of the model. In addition, the overall level of the model's central forecast was not significantly affected by exclusion of the level shift variables. Thus, exclusion had no negative consequences and resulted in appropriate estimates of the uncertainty associated with the model's forecasts.

The forecast for the quarterly inflation rate over the next 20 quarters or 5 years, as generated by the stochastic model, is shown in chart 1 (above). *Autoregressive* models such as this one produce forecasts that gradually move toward the overall mean of the data. Thus, in this instance the model forecasts that the inflation rate will gradually move from its current level of about 0.70 percent per quarter toward its historical average of 0.99 percent per quarter.

Also indicated in chart 1 are the approximate 90-percent prediction intervals associated with the forecasts for each of the next 20 quarters. Subject to the three caveats described previously, the model predicts a 90-percent certainty that the actual inflation rate for a given quarter will fall within the bounds of the forecast interval. For example, the inflation rate for the fourth quarter of 1998 is expected to fall within the range of –0.4 to 2.3 percent and this expectation should be correct nine times out of ten (again, subject to the limitations described above).

Initially, the width of the forecast interval is relatively narrow, reflecting the information made available by the stochastic model. In other words, the model provides some guidance as to the likely future inflation rates based on the recent actual levels and the past patterns of variation exhibited by the data. The probable range of variation in the immediate future is significantly smaller than the overall variation experienced in the past. The prediction intervals grow wider as the forecast horizon lengthens, however, in keeping with the reduced certainty that accompanies longer forecasts. Eventually, the width of the 90-percent forecast interval would match that of the original data.[9] At this point, the model would no longer offer additional information beyond that available from the simple statistics associated with the data. In the case of the inflation rate, this point would be reached somewhat beyond the end of the 20-quarter forecast period considered here.

It is important to note that the 90-percent intervals shown in chart 1 cannot be interpreted to mean that there is a 90-percent probability that *all* of the inflation rates for the next 20 quarters

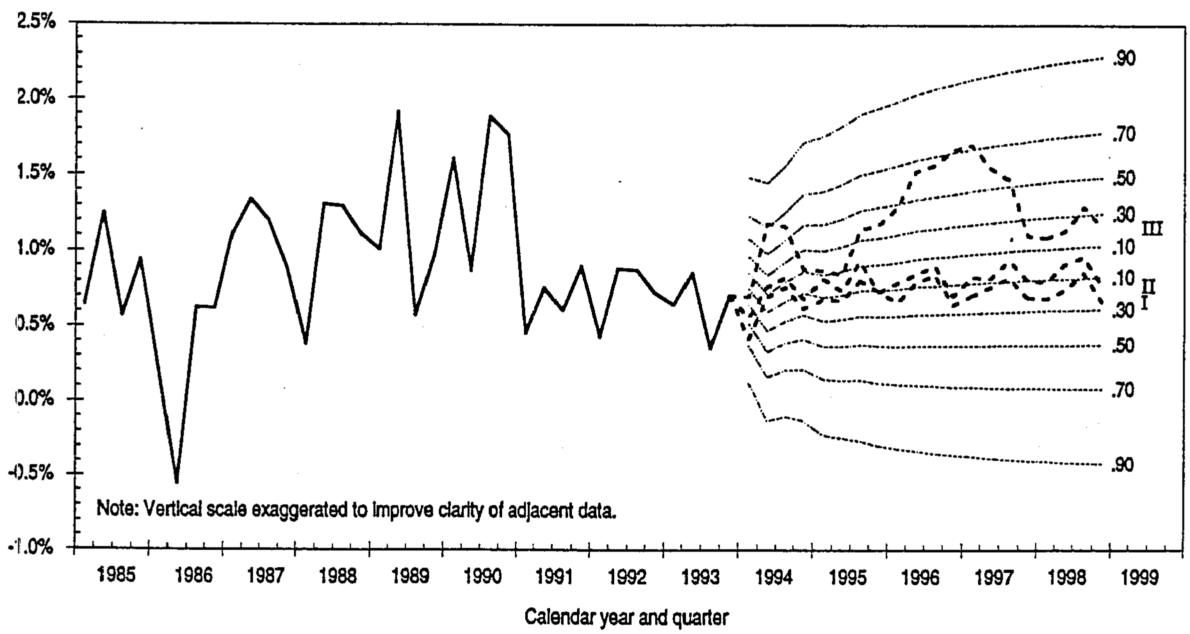
[9] The width of the 90-percent interval for the 1-quarter-ahead forecast is 1.36 percentage points. This width increases to 2.70 by the 20-quarter-ahead forecast. For comparison, the width of the 90-percent interval based on the original data is 3.08 percentage points (calculated as the difference between the 5th and 95th percentiles of the data).

will fall within the indicated bounds. The prediction interval for each quarter applies only to that quarter, irrespective of events in preceding or subsequent quarters. In practice, the probability that all of the inflation rates would fall within their respective forecast intervals is fairly small. The inflation rate is very likely to continue to exhibit substantial variation in the future and the likelihood of one or more rates falling outside of the associated prediction interval is high.

As noted previously, a number of statistical outliers and level shifts were identified for the inflation rate data but were not incorporated into the stochastic model. If these events had been incorporated, the model's central forecast would still have been very similar to the one shown in chart 1. The 90-percent prediction interval, however, would have been substantially narrower than shown because, in effect, the assumption would be made that no more unusual events would occur in the future. In practice, the not-infrequent occurrence of outliers and level shifts in the past suggests that additional ones may reasonably be expected in the future. The prediction intervals shown in chart 1 are consistent with this likelihood.

Chart 2 compares the quarterly inflation rate assumptions from the 1994 Trustees Report to a series of approximate prediction intervals calculated based on the stochastic model. Intervals are shown for certainty levels of 10, 30, 50, 70, and 90 percent. The low-cost, intermediate, and high-cost assumptions from the Trustees Report are often referred to as alternatives I, II, and III, respectively, and the latter designation is used to label chart 2.

Chart 2—Comparison of 1994 Trustees Report inflation rate assumptions and forecast intervals based on stochastic model



As indicated in chart 2, the intermediate (alternative II) inflation rate assumptions are roughly bounded by the 10-percent forecast interval. Thus, the Trustees' intermediate assumptions conform closely to the central or median forecast based on the stochastic model. In other words, for each of the next 20 quarters there is about a 50-50 probability of experiencing an inflation rate above or below the alternative II assumption. The low-cost or alternative I assumptions are generally close to the lower bound for what would be the 20-percent prediction interval. In a given quarter, the probability of experiencing an inflation rate lower than the alternative I assumption is about 40 percent based on the stochastic model.[10]

Alternatives I and II both represent "trend-average" assumptions. The inflation rate is assumed to follow a relatively steady path, deviating only to reflect minor seasonal effects. In practice, of course, future inflation rates will not be steady and will instead continue to vary upward and downward from time to time. The trend-average assumption is intended to represent the *average* future value of the economic variable and is used because of the futility of attempting to predict the specific path that will be followed in the future—predicting the average is sufficiently daunting.

The high-cost alternative III assumptions are fundamentally different in that they are based on an explicit quarter-by-quarter scenario that includes two economic recessions together with a temporary return to higher inflation. Thus, the high-cost assumptions do not represent a trend-average pattern initially; they are intended to illustrate the adverse financial effects that can result from a specific cycle of economic events. As indicated in chart 2, the alternative III assumptions generally range between the upper bound of the 10-percent prediction interval and the upper bound of the 70-percent interval.

The relative closeness of the Trustees' low-cost and intermediate assumptions may suggest that there is inadequate differentiation between the two alternatives. This is not necessarily the case since the *cumulative* probability for each of the two sets of assumptions over a number of quarters can be quite different. In other words, the probability that the inflation rate will consistently maintain a value below the median forecast year after year may be relatively low, even though this value is only slightly below the median.

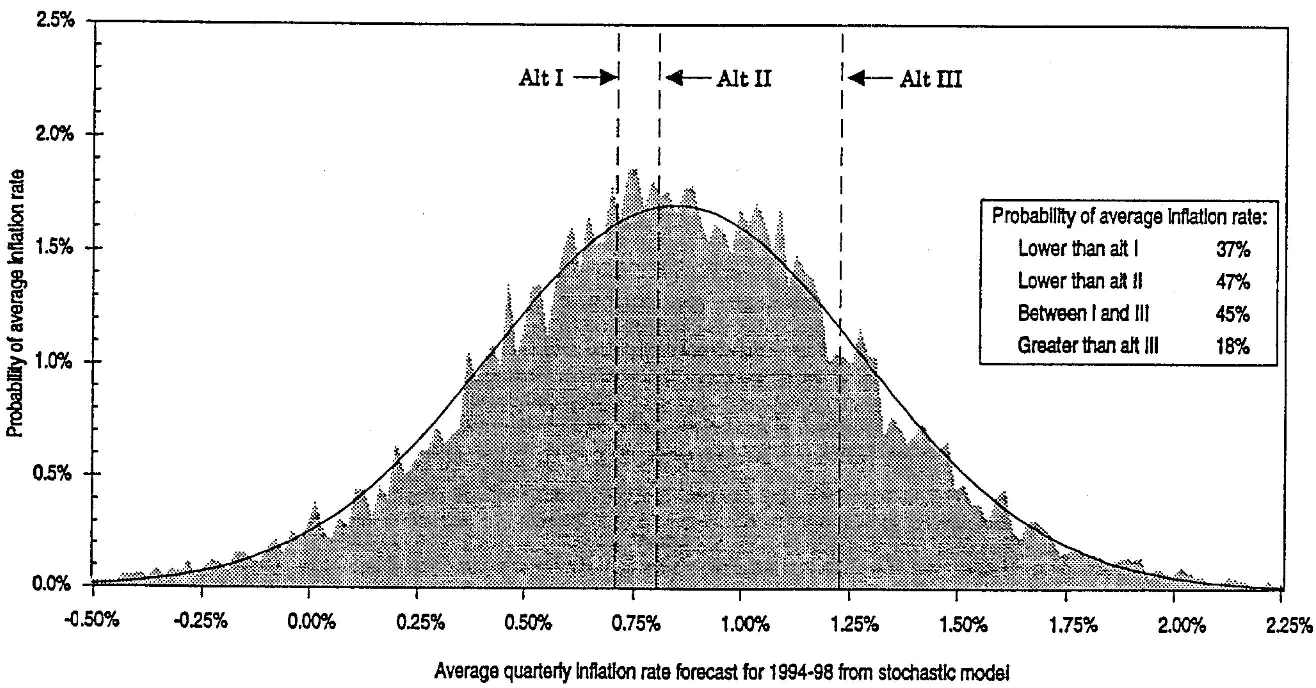
To analyze this issue further, it is instructive to compare *average* inflation rates over the next 5 years in addition to the quarter-by-quarter comparison presented in chart 2. In particular, the average inflation rate assumed under each Trustees Report alternative can be compared with a range of average forecasts based on the stochastic model. To make this comparison, the model was used to generate 5,000 individual forecast paths for the inflation rate over the 20-quarter period. Each of these "stochastically generated" forecasts reflects the patterns of variation identified by the model; that is, the inflation rate for each quarter reflects not only its relationship to past inflation rates but also chance variations of a random nature consistent with the underlying model for σ_t^2, the variance of the error term. For each of the 5,000 forecast paths, the average inflation rate was calculated for the 20 quarters. The *stochastic probability distribution* of the resulting averages is shown in chart 3. The actual distribution, based on the 5,000 sample forecasts, is shown by the gray shaded area. For ease of visualization, a smoothed probability curve is also shown.

[10] The 20-percent prediction interval represents an expectation that the actual inflation rate would be contained within the interval approximately 20 percent of the time. The other 80 percent of the time, the actual rate would fall outside the interval—40 percent of the time on the high side and 40 percent on the low side.

As one would expect, the potential range of the *average* rate of inflation over the next 5 years is estimated to be substantially narrower than the range for an individual quarter. At the low end of the distribution, the model suggests that about 5 percent of the time the average inflation rate would be less than 0.14 percent. At the opposite extreme, the model forecasts that 5 percent of the time the inflation rate would average 1.59 percent per quarter or more (equivalent to 6.5 percent on an annual basis). The other 90 percent of the time, inflation would be expected to fall somewhere in-between these figures. The midpoint of the inflation rate distribution is 0.85 percent per quarter (3.4 percent per year).

The average quarterly inflation rates assumed under alternatives I, II, and III of the 1994 Trustees Report are 0.71, 0.81, and 1.22 percent, respectively. These values are shown superimposed on the distribution in chart 3. The alternative II assumption of 0.81 percent inflation per quarter falls on about the 47th percentile of the stochastic forecast distribution—in other words, fairly close to the median. The low-cost (alternative I) assumption average corresponds to the 37th percentile, indicating a 37-percent probability of a lower inflation rate than assumed and a 63-percent probability of a higher rate. The alternative III assumption, following an explicit "high-cost" path, falls on the 82nd percentile; approximately 18 percent of the model's average forecasts exceed this assumption.

Chart 3—Stochastic probability distribution of average quarterly inflation rate and comparison to Trustees Report assumption averages



These results suggest that there is a 45-percent probability that inflation will average between 0.71 and 1.22 percent per quarter over the next 20 quarters—the range delineated by the alternative I and III assumptions from the 1994 Trustees Report. The model forecasts further suggest that the intermediate alternative II assumptions are relatively close to the median expectation. There is about a 47-percent chance of lower inflation and about a 53-percent chance of higher inflation. The validity of these interpretations, naturally, remains dependent on the conditions described previously concerning the proper specification and estimation of the model and whether the future will behave similarly to the past.

These results do not indicate whether the width of the alternative I to III range is appropriate. If the financial outlook for the Social Security program depended solely on future inflation rates, then a range of assumptions encompassing only 45 percent of the possible outcomes would seem unduly narrow. In reality, of course, inflation is only one factor among a large number that will influence the future financial operations of the program. The likelihood of *all* factors simultaneously differing in the same direction from their central values is necessarily lower than for any individual factor alone (although not necessarily uncommon). This is an important question to answer in evaluating the Trustees Report assumptions but one that exceeds the scope of this report. Further research will be required, employing multivariate statistical techniques, econometric models, and/or other approaches. As noted in the introduction, this study is a first attempt at using statistical time series techniques to help evaluate the range of variation present in the Trustees Report assumptions for selected economic variables considered individually.

III. Unemployment Rate

Over the next 5 years, the level of unemployment in the U.S. will have an important effect on the financial status of the Social Security program. The primary source of financing for the trust funds is the Social Security taxes paid by covered employees, employers, and self-employed individuals. Periods of relatively high unemployment correspond directly to fewer taxpayers (and also contribute to increased incidence of early retirement and higher levels of applications for disability benefits). The high rates of unemployment experienced during the 1974-75 and 1980-82 recessions contributed significantly to the OASDI financing crises experienced at those times.

Chart 4 plots past values of the unemployment rate, starting with the first quarter of 1950 and extending through the last quarter of 1993.[11] The rate of unemployment shows a pattern of cyclical variation corresponding to the business cycle, with peaks associated with the major economic recessions experienced since 1950 and troughs associated with the subsequent periods of recovery. The data also appear to fluctuate around different average levels before and after the mid-1970s, with the later average significantly exceeding the earlier one.

Chart 4—Quarterly unemployment rate for 1950-93 and forecast for 1994-98 with 90-percent forecast interval

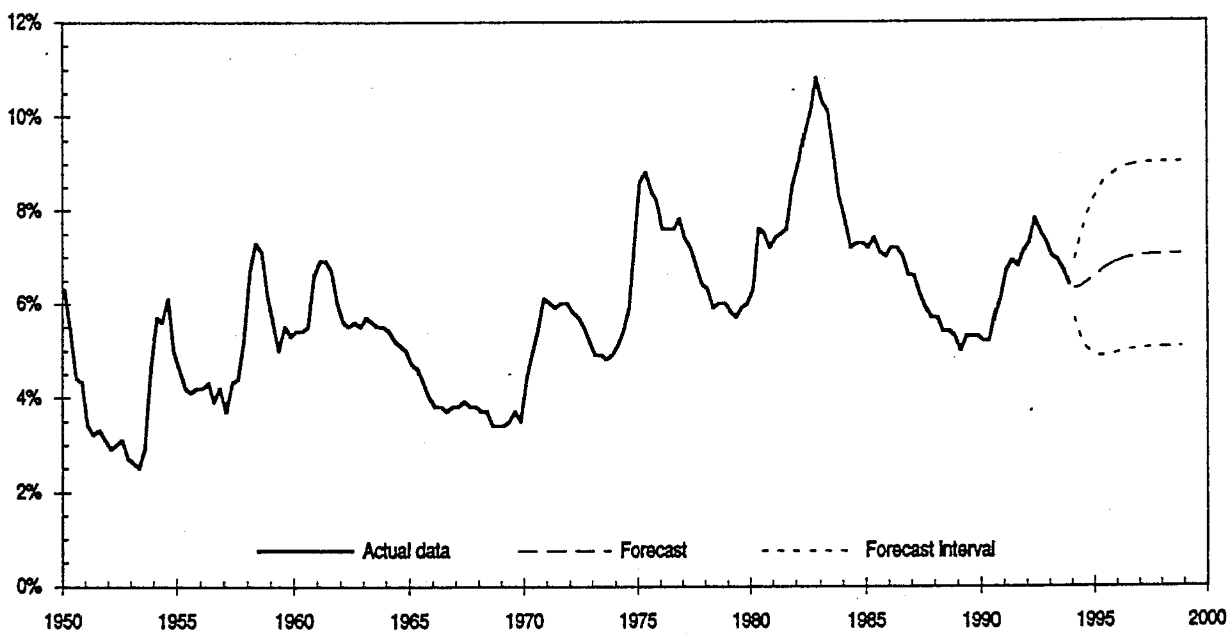


Table 2 presents summary statistics for the unemployment rate data. In addition to statistics for the full period 1950-93, figures are shown for two subperiods corresponding to the apparent change

[11] The data shown are based on the seasonally adjusted rate of unemployment among the civilian labor force. The data and forecasts are consistent with the Bureau of Labor Statistics' pre-1994 methodology for measuring unemployment and thus do not reflect their latest survey definitions.

in the average level of unemployment. As indicated, the quarterly unemployment rate averaged 5.8 percent during 1950-93, with a standard deviation of 1.6 percent. However, the average prior to 1974 (4.8 percent) is significantly different than the subsequent average (7.0 percent) and the standard deviation for each subperiod is substantially less than for the data overall. Within each subperiod, a range of roughly 1.8 percentage points encompasses the middle 50 percent of the unemployment data (measured as the difference between the 25th and 75th percentiles). Thus, the summary statistics tend to confirm the impression given by chart 4 that the average level of unemployment shifted in 1974. The statistics also suggest that the pattern of variation in the early period about the pre-1974 average is fairly similar to the pattern shown in the later period about the later average.

Table 2—Summary statistics for unemployment rate

Period	Number of points	Mean	Standard deviation	Minimum value	25th percentile	50th percentile	75th percentile	Maximum value
1950-93...	176	5.8%	1.6%	2.5%	4.6%	5.7%	7.0%	10.8%
1950-73...	96	4.8%	1.1%	2.5%	3.8%	4.9%	5.6%	7.3%
1974-93...	80	7.0%	1.3%	5.0%	5.9%	7.0%	7.6%	10.8%

As was the case with the inflation rate, the appearance of the data in chart 4 suggests that a high rate of unemployment in one quarter tends to be followed by additional high rates, and vice-versa. The time series model established for the unemployment rate is shown below. A detailed discussion of its derivation and characteristics is available in appendix C.

$$u_t = 0.4749 + 1.3936 u_{t-1} - 0.4931 u_{t-2} + 0.2262\, LS_{74:I} + \varepsilon_t$$

$$\text{where } LS_{74:I} = \begin{cases} 0 \text{ for } t < 1974\text{:I} \\ 1 \text{ for } t \geq 1974\text{:I} \end{cases}$$

As is the case with the other time series models in this study, this is an autoregressive formula that models the economic variable at time t as a weighted average of the corresponding variables in one or more prior periods. In this case, the unemployment rate u_t, is related to the unemployment rates in the prior 2 quarters (u_{t-1} and u_{t-2}). As before, the factor ε_t represents a random shock or error affecting the unemployment rate each quarter. In contrast to the model for the inflation rate, the distribution for the unemployment error term is assumed to have a constant variance through time (see appendix C for details).

The model for the unemployment rate also includes a term to reflect the shift in the level of unemployment rates starting in 1974. This term was included as a result of a formal search for statistical outliers and level shifts performed using special identification techniques developed by researchers in recent years. The identification process confirmed the existence of a statistically significant shift in the level of the unemployment rate starting in 1974.[12] The nature of statistical outliers and level shifts is described in appendix A and the specific results associated with the unemployment rate model are shown in appendix C. In the stochastic model, the "indicator" or "dummy" variable $LS_{74:I}$ takes the value of 0 prior to 1974 and 1 in 1974:I and later. The effect of this variable, in conjunction with the other parameters of the model, is to gradually increase the estimated average level of unemployment from about 4.8 percent prior to 1974 to about 7.0 percent thereafter. In the absence of this variable, the model would estimate the average level of unemployment at a constant 5.8 percent throughout the historical period. As indicated in appendix C, however, such a model leads to forecasts that are inconsistent with the current level of the data and to anomalies with the forecast intervals.

Economists have hypothesized that a number of demographic and structural factors have contributed to higher rates of unemployment starting in the 1970s. The combination of the oil price inflations of the mid- and late-1970s, entrance of the "baby boom" generation into the labor force, participation of a larger proportion of women in the paid labor force, withdrawal of U.S. troops from Vietnam, shifts from industrial production toward services, increasing job skill requirements, restructuring and streamlining of businesses, and other factors may well have had a substantial, ongoing effect on the overall level of unemployment. An econometric analysis of the effects of these factors is beyond the scope of this study. For purposes of forecasting economic conditions for the next 5 years, however, there seems to be little reason to expect a return to a significantly lower level of unemployment than has been experienced over the last 2 decades.[13]

The forecast for the unemployment rate over the next 20 quarters, generated by the stochastic model, is shown in chart 4. As noted in the preceding section, autoregressive models produce forecasts that gradually move toward the overall mean of the data. When level shifts are incorporated, the forecast moves toward the average rate for the current (that is, most recent) level of the data. Thus, in this instance the model forecasts that the unemployment rate will gradually move from 6.4 percent at the end of 1993 toward its post-1973 average of 7.0 percent.

Also indicated in chart 4 are the 90-percent prediction intervals associated with this forecast for each of the next 20 quarters. For example, the model predicts that the actual unemployment rate for the fourth quarter of 1998 will fall within the range of 5.1 to 9.0 percent and that this expectation should be correct nine times out of ten. As always, forecasts of this nature are subject to the three caveats described in the introduction to this study concerning (1) quantum changes in socioeconomic circumstances, (2) appropriateness of model specification, and (3) accuracy of model estimation.

As seen with the model for inflation rates, the width of the forecast interval widens as the forecast horizon lengthens, in keeping with the reduced certainty that accompanies longer forecasts. By the end of the 5-year forecast, the width of the 90-percent interval nearly matches that of the orig-

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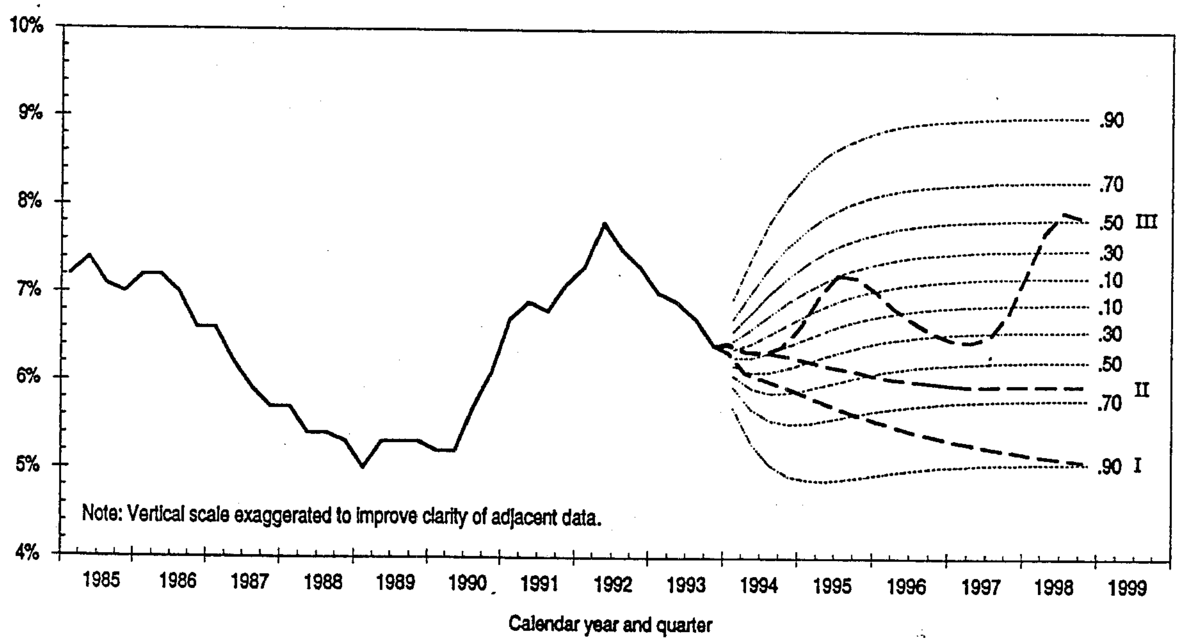
[12] Other research also supports the existence of such a shift. See, for example, Evans [37], Perron [50], and Weiner [54].

[13] The effect of such factors on the "natural" rate of unemployment during 1961-93 and a discussion of future prospects are available in a recent article by Stuart Weiner of the Kansas City Federal Reserve [54].

inal data.[14] As before, the 90-percent probability associated with the intervals shown in chart 4 applies to the expected unemployment rate for a given quarter and does not represent the probability that *all* of the unemployment rates for the next 20 quarters will fall within the indicated bounds.

Chart 5 compares the quarterly unemployment rate assumptions from the 1994 Trustees Report to the 10-, 30-, 50-, 70-, and 90-percent prediction intervals based on the stochastic model. As indicated, the unemployment rates assumed under alternative II decline for several years and then level off at about the lower bound for the 60-percent interval (midway between the displayed lower bounds for the 50- and 70-percent intervals). In this instance, the intermediate assumptions are significantly lower than the median forecast based on the stochastic model. At the end of the forecast period, for example, the model suggests there is about a 20-percent probability of unemployment being lower than alternative II and about an 80-percent probability of it being higher.[15] If the stochastic model excluded the variable representing the level shift in the data in 1974, then the resulting forecast intervals would be much more centered relative to the alternative II assumptions. As noted, however, the statistical criteria used in selecting a model point strongly to the need to incorporate the level shift. In addition, there is little economic evidence to suggest that a return to pre-1974 levels of unemployment is imminent.

Chart 5—Comparison of Trustees Report unemployment rate assumptions and forecast intervals based on stochastic model



[14] The width of the 90-percent prediction interval for the 1-quarter-ahead forecast is 1.23 percentage points. This width increases to 3.95 by the 20-quarter-ahead forecast. For comparison, the width of the 90-percent interval based on the original data is 4.30 percentage points.

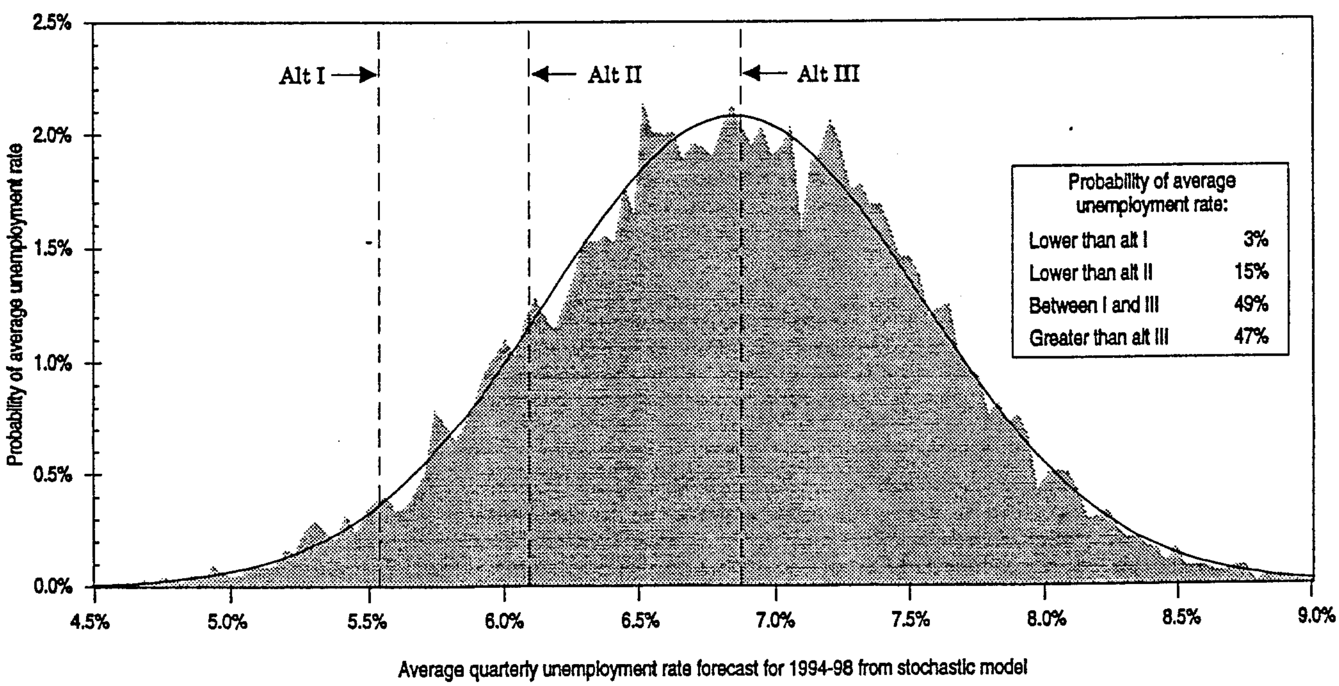
[15] The 60-percent prediction interval represents an expectation that the actual inflation rate would be contained within the interval approximately 60 percent of the time. The other 40 percent of the time, the actual rate would fall outside the interval—20 percent of the time on the high side and 20 percent on the low side.

The low-cost assumptions (alternative I) follow a similar, but lower, path and end up at the lower bound for the 90-percent forecast interval. In other words, using the stochastic model as the criteria, at the end of the 5-year period there would be only about a 5-percent probability of experiencing unemployment rates lower than assumed by alternative I.

As noted in conjunction with the inflation rate assumptions, alternatives I and II both represent trend-average assumptions and follow a relatively steady path. In contrast, the high-cost alternative III assumptions are based on the explicit "twin recession" scenario described previously. As indicated in chart 5, the alternative III assumptions follow a cyclical path that varies from the upper bound for the 50-percent forecast interval, on the high side, to the lower bound for the 30-percent interval on the low side. *If* the stochastic model is an appropriate guide to the possible range of future variation in the unemployment rate, then the alternative III assumptions, with unemployment peaks of 7.2 and 7.9 percent, do not appear to represent a particularly adverse scenario. As noted in the Trustees Report, "The total declines in real GDP [gross domestic product] for the two projected recessions are slightly less than those of recent recessions; however, the duration of recovery between these recessions is assumed to be much shorter than for recoveries experienced in the past 2 decades." This feature of the scenario tends to increase the degree of pessimism and to help offset the effect of the relatively low peak unemployment rates.

Chart 6 compares the Trustees Report unemployment assumptions to the forecasts from the stochastic model on the basis of the average level for 1994 through 1998. As before, to make this comparison the model was used to generate 5,000 individual forecast paths for the unemployment rate over the 20-quarter period. Each of these forecasts reflects the relationship between current

Chart 6—Stochastic probability distribution of average quarterly unemployment rate and comparison to Trustees Report assumption averages



and prior unemployment rates as well as chance variations of a random nature. The average future unemployment rate was calculated for each forecast and the distribution of these averages is shown below by the gray shaded area, together with the smoothed probability curve.

As expected, the potential range of the average rate of unemployment over the next 5 years is estimated to be significantly narrower than the range for an individual quarter. The 90-percent interval for the average rate is 5.7 to 8.0 percent (in contrast to the corresponding range of 5.1 to 9.0 percent seen previously for the estimated unemployment rate in the fourth quarter of 1998). The median of the average unemployment rate distribution is 6.8 percent.

The average unemployment rates assumed under alternatives I, II, and III of the 1994 Trustees Report are about 5.5, 6.1, and 6.9 percent, respectively. These values are shown superimposed on the distribution in chart 6. The alternative II assumption of 6.1 percent corresponds to only the 15th percentile of the stochastic forecast distribution—in other words, significantly below the median. The alternative I average corresponds to the 3rd percentile, indicating only a 3-percent probability of a lower average unemployment rate than assumed over the 5-year period. The alternative III assumption falls on the 47th percentile, very close to the median of the stochastic forecasts.

Based on the estimated forecast distribution, there is about a 49-percent probability that the average unemployment rate over the next 20 quarters will fall within the range delineated by the alternative I and III assumptions from the 1994 Trustees Report. The probability of this "included range" is fairly close to that seen previously for the inflation rate assumptions. Once again, however, the univariate statistical analysis offers little guidance concerning an appropriate width for the range of Trustees Report assumptions.

The model results also indicate that the location of the alternative I-to-III range within the stochastic distribution is significantly off-center. This finding suggests that more investigation is advisable into the location of the intermediate and high-cost assumptions. If the model results are reasonable, then the Trustees Report assumptions for the unemployment rate may be too optimistic.

As before, these conclusions are subject to the three conditions described in section I.

IV. Real Interest Rate

The financial status of the Social Security program can be affected by the level of interest earned on the invested assets of the OASI and DI Trust Funds. The magnitude of the effect depends on the relative level of trust fund assets. During much of the program's history, assets were fairly low in relation to expenditures, with the result that interest was not a significant factor in trust fund income. In more recent years, OASDI assets (in aggregate) have been growing rapidly and are expected to continue to do so for roughly another 20 years. As asset levels continue to increase, interest income to the program will become a larger proportion of total program income.[16] A key determinant of the amount of future interest earnings will be the interest rates payable on new trust fund securities.

The assets of the Social Security trust funds are invested primarily in special Treasury securities issuable only to the trust funds.[17] By law, the interest rate payable on new securities issued in a given month is set equal to the current yield on all marketable Federal securities that are not due to mature for at least 4 years. The nominal interest rate payable on new securities issued in December 1993 was 5 7/8 percent, compounded semiannually. For purposes of evaluating trust fund financial status, it is generally more instructive to study *real* interest rates—the rate of interest earned after adjustment for inflation. Thus, the focus of this section will be on real interest rates, defined loosely as the nominal interest rate minus the rate of increase in inflation (see appendix D for a more detailed description of the data).

Although this study focuses on the economic factors affecting Social Security in the short range (1994-98), it is important to note the role played by the real interest rate in evaluating the *long-range* financial status of the program. The "actuarial balance" for OASDI depends primarily on the difference between the discounted present values of future tax income and future expenditures over the next 75 years. These present value calculations are very sensitive to assumed future real interest rates which, in effect, act as a weighting mechanism. The higher the assumed real interest rate, the less weight given to more distant future income and expenditure amounts. Since the OASDI program is projected to experience large surpluses for a number of years, followed by large deficits thereafter, the weighting applied to these periods can significantly affect the resulting actuarial balance. Table II.G6 of the 1994 Trustees Report provides an analysis of the sensitivity of the long-range actuarial projections to assumed real interest rates.

Chart 7 plots past values of the real interest rate, starting with the first quarter of 1961 and extending through the last quarter of 1993.[18] During 1961-72, real interest rates stayed in the neighborhood of 2 percent. In the mid-1970s and again in the late 1970s, major increases in inflation caused real rates to plummet. With the deflationary recession of the early 1980s, real interest

[16] In 1993, interest represented 7.8 percent of total income to the OASI and DI Trust Funds. Roughly 13 percent of income is estimated to arise from interest by the year 2010.

[17] At the end of 1993, 99.98 percent of the funds' assets were in such securities, with the remainder in "marketable" Treasury securities (that is, securities available to the general public and traded on the open market). Investments are also allowed in securities guaranteed by the Federal government, such as those issued by the Federal National Mortgage Association, but none are currently held by the trust funds.

[18] Current trust fund investment procedures were first implemented in October 1960. Prior to then, interest rates were determined by Congress, rather than by market forces, and thus were not relevant to this analysis.

rates experienced an extraordinary increase to as much as 9 percent, followed by a gradual decline to below 4 percent during 1984-91.

In addition to the reduced rate of inflation, several other factors may have contributed to the extraordinary increase in real interest rates in the early 1980s. These include (i) the change in Federal Reserve policy toward governing money supply growth rather than interest rates (adopted in October 1979 and phased out in 1982-87); (ii) the rapid growth in Federal budget deficits in the early 1980s and the corresponding increase in Federal borrowing; (iii) reduced savings rates in the private sector; and (iv) the deregulation of banks and other financial institutions that occurred in the early 1980s. As before, an econometric analysis of the effects of these factors is beyond the scope of this study.

Chart 7—Quarterly real interest rate for 1961-93 and forecast for 1994-98 with 90-percent forecast interval

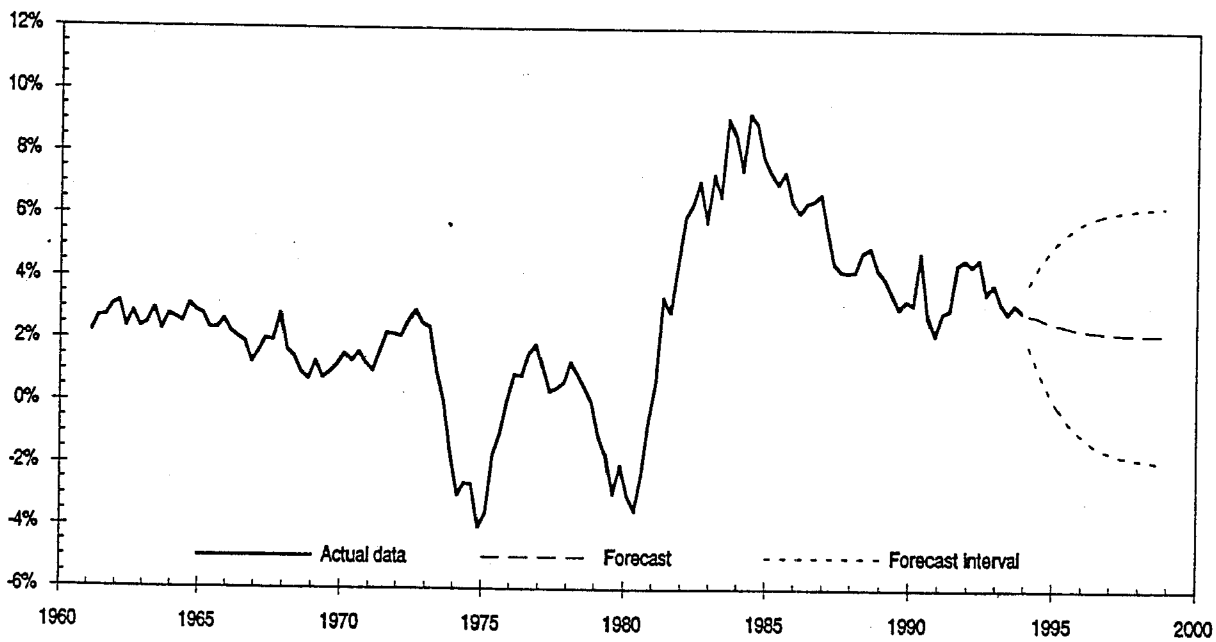


Table 3 indicates that the real interest rate averaged 2.6 percent during 1961-93, with a standard deviation of 2.8 percent. Although the range of variation over the whole period was quite wide, from as low as –4 percent to more than 9 percent, one-half of the rates were in the range of about 1.0 to 4.2 percent.

Table 3—Summary statistics for real interest rate

Period	Number of points	Mean	Standard deviation	Minimum value	25th percentile	50th percentile	75th percentile	Maximum value
1961-93...	132	2.58%	2.78%	–4.02%	1.02%	2.54%	4.22%	9.27%

As was the case with the inflation and unemployment rates, the appearance of the data in chart 7 suggests that the real interest rate in one quarter is likely to be similar in magnitude to values in prior quarters. The time series model established for this data series is shown below. The model estimates the real interest rate at time t (denoted ri_t) as a function of the rates in the prior quarter and the fourth prior quarter (ri_{t-1} and ri_{t-4}). A detailed discussion of the model's derivation and characteristics is available in appendix D.

$$ri_t = 0.1731 + 1.0765\,ri_{t-1} - 0.1534\,ri_{t-4} + 2.9072\,IO_{83:III} + \varepsilon_t$$

$$\text{where } IO_{83:III} = \begin{cases} 0 \text{ for } t \neq 1983\text{:III} \\ 1 \text{ for } t = 1983\text{:III} \end{cases}$$

$$\text{and where } \sigma_t^2 = 0.0657 + 0.1967\,\varepsilon_{t-1}^2 + 0.6889\,\sigma_{t-1}^2$$

The model also includes a term to reflect the period of extremely high real interest rates in the early 1980s. This term was included as a result of the formal search for statistical outliers and level shifts performed for each of the economic series analyzed in this study (see appendix A). For the real interest rate data, the procedure identified a statistically significant *innovational outlier* in the third quarter of 1983. Such outliers are characterized by a sudden large increase (or decrease) in the level of the series, followed by a gradual transition back to the series average. The nature of this outlier and the reasons for incorporating it into the model are described more fully in appendix D. In the stochastic model, the indicator variable $IO_{83:III}$ takes the value of 0 for all quarters other than 1983:III and the value 1 in 1983:III. The effect of this variable, in conjunction with the other parameters of the model, is to increase the estimated real interest rate by roughly 2.9 to 3.6 percentage points for a few quarters, starting in 1983:III, with the differential declining gradually thereafter.

As usual, the error term ε_t reflects the element of residual random variation in the real interest rate each quarter. In this instance, as was the case for the inflation rate, it was necessary to allow the statistical variance of the error term to change over time. Consequently, the auxiliary equation shown above is used for estimating the variance at time t, designated as σ_t^2, as a function of the variance in the prior quarter, σ_{t-1}^2, and the error term in the prior quarter ε_{t-1}.

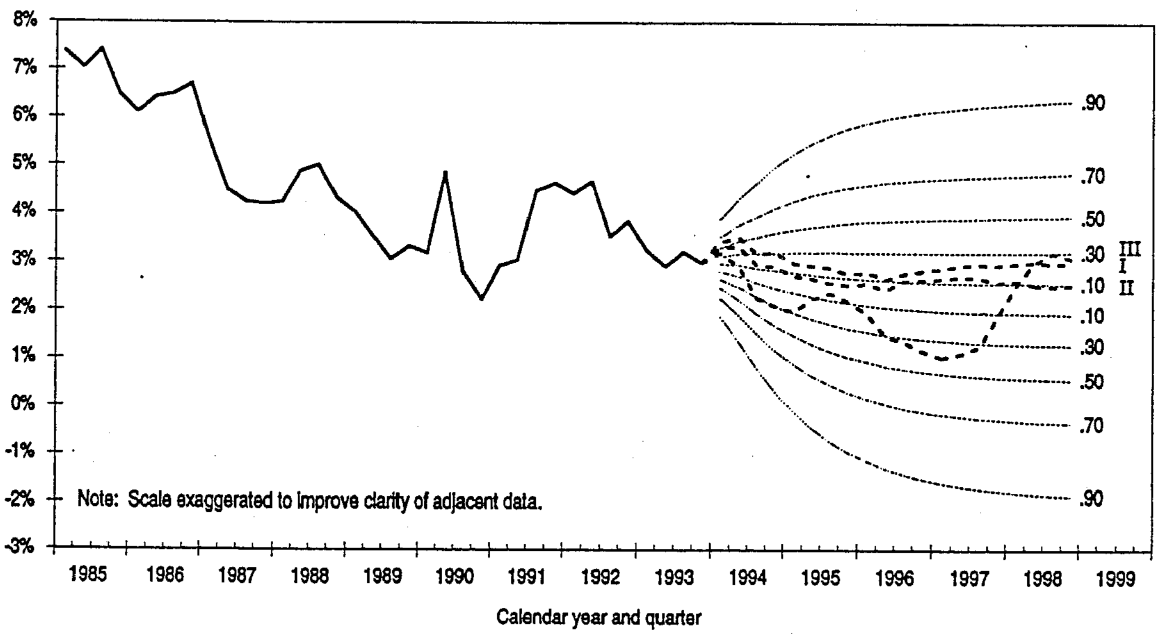
The short-range forecast produced by the real interest rate model is shown above in chart 7. The forecast gradually moves toward the overall estimated mean of the data, in keeping with the properties of autoregressive models. Incorporating the innovational outlier into the model resulted in an estimated mean level that is slightly lower than it would have been in the absence of the outlier adjustment. For purposes of forecasting interest rates for the next 5 years, there seems to be little reason to expect a recurrence of the unusual confluence of factors that contributed to the exceptionally high real interest rates in the early 1980s.

Also indicated in chart 7 are the approximate 90-percent intervals associated with the short-range forecast. The real interest rate for the fourth quarter of 1998, for example, is estimated to fall within the range of –1.9 to 6.4 percent with a certainty of 90 percent. As usual, this forecast is subject to the three important caveats described in the introduction to this study concerning limitations of stochastic models.

The 90-percent prediction interval may appear unusually wide at the end of the forecast period, especially in comparison to the relatively narrow range of rates experienced during the last few years. On the other hand, the actual past peaks and troughs are considerably outside the range of the prediction interval. This result reflects something of a compromise by the stochastic model. First, the current estimate of the error term variance, σ_{ϵ}^{2}, is relatively low (reflecting the recent stability of real interest rates). Overall, however, the model reflects the strong autoregressive nature of interest rates and the possibility that they may again wander substantially from their average level. Thus, the prediction intervals widen rapidly for future quarters in recognition of the increased longer-term uncertainty.[19] As before, the 90-percent probability associated with the intervals shown in chart 7 applies to the expected real interest rate for a given quarter only, not to all 20 quarters collectively.

Chart 8 compares the quarterly real interest rate assumptions from the 1994 Trustees Report to the series of approximate forecast intervals based on the stochastic model. As indicated, the alternative II real interest rates are very close to the upper bound of the 10-percent interval throughout the forecast period. As such, alternative II is fairly close to the central or median forecast based on

Chart 8—Comparison of Trustees Report real interest rate assumptions and forecast intervals based on stochastic model



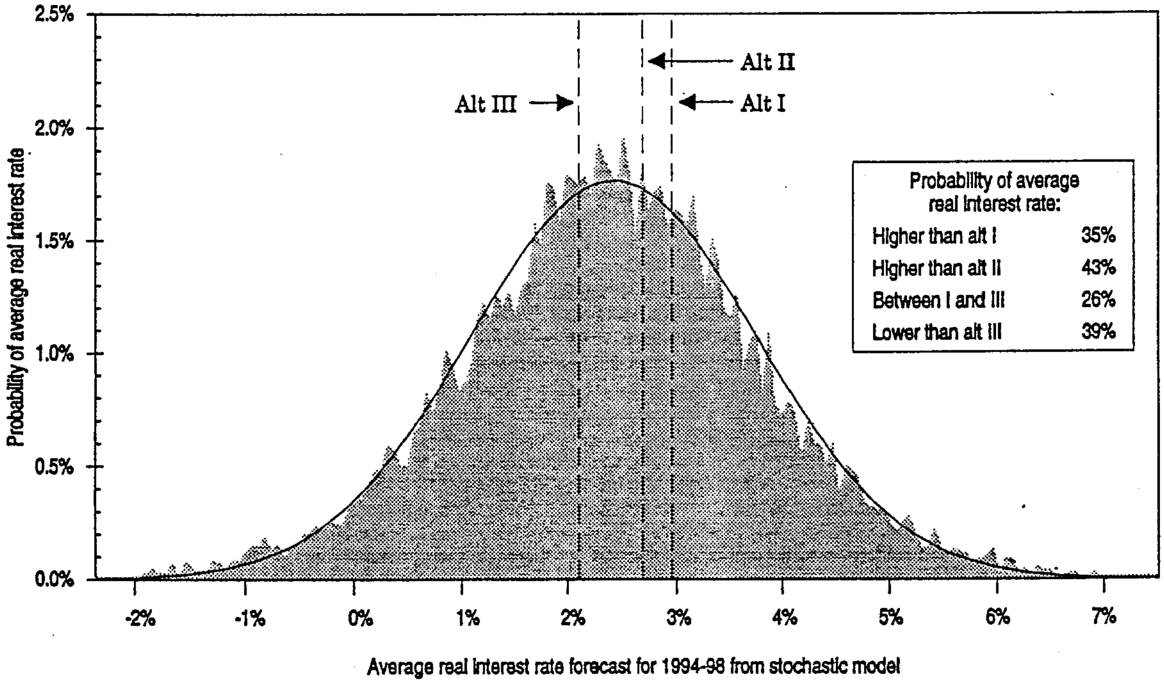
[19] As seen previously for the inflation and unemployment rate models, the width of the 90-percent prediction interval eventually approaches that of the original data. The width of the prediction interval for the 1-quarter-ahead forecast is 2.0 percentage points and increases to 8.2 at the 20th quarter. The width of the 90-percent interval based on the original data is 10.2 percentage points.

the stochastic model. The more optimistic alternative I assumptions are very close to alternative II, generally falling near the upper bound of the 20-percent interval (not shown). As noted in previous sections, alternatives I and II both represent trend-average assumptions.

The high-cost alternative III assumptions, based on the "twin recession" scenario, vary widely—from the lower bound for the 40-percent prediction interval to the upper bound for the 30-percent interval. Most of this variation is attributable to the underlying inflation assumptions (described in section II of this study). Overall, the range of variation encompassed by the Trustees Report alternatives appears to be somewhat narrower, in relation to the model results, than was the case for either the inflation or unemployment assumptions.

Chart 9 compares the average future real interest rates assumed in the Trustees Report for 1994-98 with a distribution of 5,000 individual forecast paths based on the stochastic model.

Chart 9—Stochastic probability distribution of average real interest rate and Trustees Report assumption averages



The potential range of the average rate of real interest over the next 5 years is estimated to be 0.1 to 4.7 percent (based on a confidence level of 90 percent). This range is substantially narrower, of course, than the corresponding range of –1.9 to 6.4 percent seen previously for the estimated real interest rate in the fourth quarter of 1998. The median of the stochastic distribution for the average real interest rate is 2.5 percent.

The average real interest rates assumed for 1994-98 under alternatives I, II, and III are about 3.0, 2.7, and 2.1 percent, respectively. The alternative II assumption corresponds to about the 57th percentile of the stochastic forecast distribution—slightly above the median. The average real interest rate assumed under alternative I corresponds to the 65th percentile, while the average for alternative III is at the 39th percentile.

These results suggest that there is only about a 26-percent probability that the average real interest rate will fall within the range of 2.1 to 3.0 percent over the next 20 quarters, as assumed by the 1994 Trustees Report. This probability is significantly narrower than those seen previously for the inflation and unemployment assumptions. The location of the alternative I-to-III range, however, is reasonably well centered within the stochastic distribution.

The validity of these interpretations, as always, is dependent on the proper specification and estimation of the model and on the assumption that the future will behave similarly to the past. In addition, the model results do not provide a means for judging whether an appropriate range of variation is assumed in the Trustees Report for the real interest rate. The results suggest, however, that the assumed range is narrower—relative to the variation exhibited in the past—than has been assumed for inflation and unemployment rates. For consistency with the range of variation estimated to be present in the other Trustees Report assumptions, it may be considered desirable to broaden the assumption range for the real interest rate.

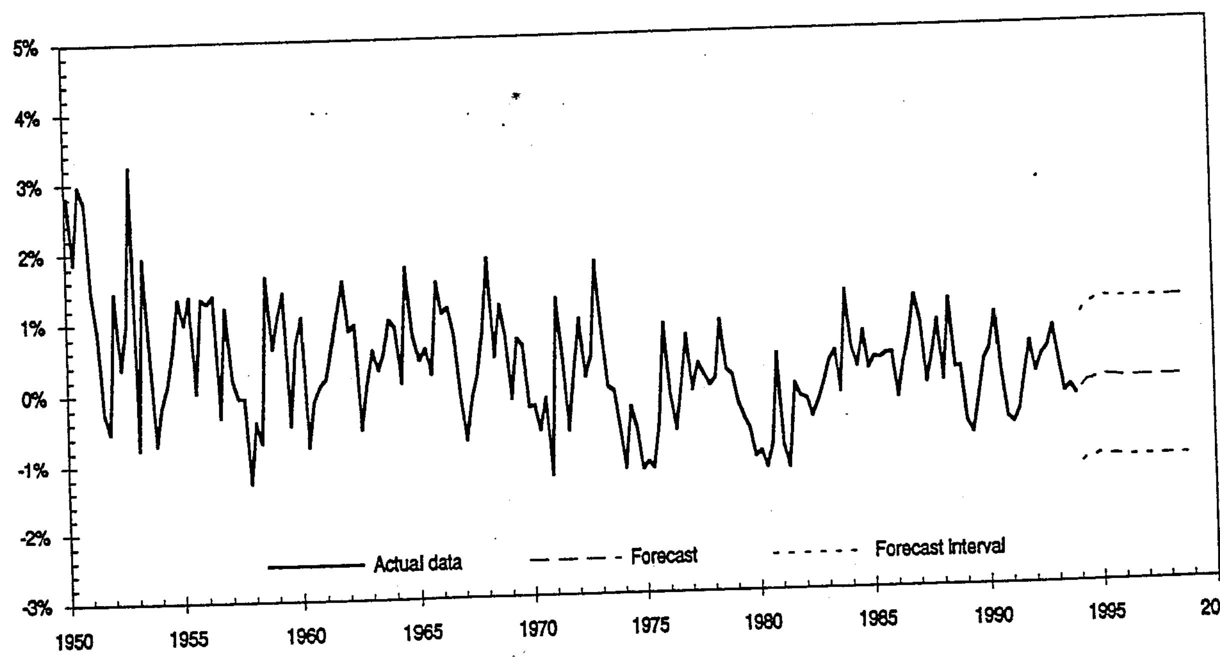
V. Real Wage Increase

One of the most sensitive determinants of Social Security's short-range financial condition is the balance between wage increases and price increases. Social Security payroll taxes increase in direct proportion to increases in the earnings of workers. Benefits increase as a function both of cost-of-living adjustments and increases in average earnings. As a general rule, wage increases lead immediately to increased tax income and, more gradually, to increases in benefits. Price increases, if not matched by contemporaneous wage increases, result in benefit increases that exceed the growth in tax income. The program's long-range financial status is also strongly influenced by real wage growth, as indicated by the sensitivity analysis shown in table II.G4 of the 1994 Trustees Report.

In the annual Trustees Report, the balance between wage and price increases is measured by the *real wage differential*. This factor is defined as the annual percentage increase in the average wages of employees covered under the Social Security program, minus the corresponding increase in the Consumer Price Index. Positive values indicate a favorable financial effect, with taxes increasing more rapidly than benefits, thereby improving the financial position of the trust funds. Negative values, conversely, indicate an adverse financial effect.

As noted above, the real wage differential is defined and calculated on an annual basis. For technical reasons, it was necessary to perform the statistical analysis of real wage growth using *quarterly* data. Accordingly, a quarterly data series was constructed using the same data sources, to the extent possible, that underlie the real wage differential.[20] The resulting series is referred to in this study as the quarterly real wage increase and is shown in chart 10.

Chart 10—Quarterly real wage increase for 1950-93 and forecast for 1994-98 with 90-percent forecast interval



[20] The reasons for developing the quarterly series and the methods used in doing so are described in appendix E. A comparison of the quarterly series with the real wage differential is also provided.

As indicated in chart 10, throughout the period 1950-93 the rate of increase in real wages has tended to vary substantially from one quarter to the next. Furthermore, in contrast to the results seen for the inflation rate, unemployment rate, and real interest rate, there is little indication that the increase in real wages in one quarter is strongly related to the increases in preceding quarters. Apart from its apparent randomness, the appearance of the data also suggests that the average level of the quarterly real wage increase may have shifted downward starting in about the mid-1970s.

Table 4 presents summary statistics for the quarterly real wage increase in 1950-93 and for two subperiods corresponding to the apparent change in the average level of the data. The increase in real wages averaged 0.29 percent per quarter during 1950-93 (equivalent to about 1.2 percent on an annual basis) and had a standard deviation of 0.83 percent. The average prior to 1973:II (0.59 percent) was significantly different from the subsequent average (–0.06 percent). Thus, the summary statistics support the impression given by chart 10 that the average level of the real wage increase shifted in 1973. The statistics also indicate that the range of variation in the first subperiod was somewhat greater than in the second.

Table 4—Summary statistics for quarterly real wage increase

Period	Number of points	Mean	Standard deviation	Minimum value	25th percentile	50th percentile	75th percentile	Maximum value
1950-93	176	0.29%	0.83%	–1.28%	–0.26%	0.20%	0.80%	3.22%
1950-73:I	93	0.59%	0.90%	–1.28%	–0.09%	0.59%	1.16%	3.22%
1973:II-93 ...	83	–0.06%	0.59%	–1.22%	–0.49%	–0.03%	0.30%	1.24%

The stochastic model established for this data series is shown below. The model estimates the quarterly real wage increase at time t (denoted rw_t) as a function of the increases in the prior 4 quarters (rw_{t-1} through rw_{t-4}) and of two indicator variables, one to reflect the level shift described above and one to adjust for a significant outlier that occurred in the fourth quarter of 1952. A detailed discussion of the model's derivation and characteristics is available in appendix E.

$$rw_t = 0.3534 + 0.2138\,rw_{t-1} + 0.1437\,rw_{t-2} + 0.0954\,rw_{t-3} - 0.2017\,rw_{t-4}$$

$$+ 2.3604\,AO_{52:IV} - 0.4011\,LS_{73:II} + \varepsilon_t$$

$$\text{where } AO_{52:IV} = \begin{cases} 0 \text{ for } t \neq 1952:IV \\ 1 \text{ for } t = 1952:IV \end{cases} \quad \text{and} \quad LS_{73:II} = \begin{cases} 0 \text{ for } t < 1973:II \\ 1 \text{ for } t \geq 1973:II \end{cases}$$

The indicator variables were included as a result of the standard search for statistical outliers and level shifts (described in appendix A). For the quarterly data on real wage increases, the procedure identified a statistically significant *additive outlier* in 1952:IV. Additive outliers are characterized by a sudden large increase (or decrease) in the level of the series in one quarter only.[21] The search also formally identified the apparent level shift noted above, with 1973:II as the beginning quarter (interestingly, about the same time as the level shift in unemployment rates). The reasons for incorporating these statistical abnormalities into the model are described more fully in appendix E. In the model, the indicator variable $AO_{52:IV}$ takes the value of 0 for all quarters other than 1952:IV and the value 1 in 1952:IV. The effect of this variable is to increase the estimated real wage increase by about 2.4 percentage points in 1952:IV (and by relatively small amounts for another few quarters thereafter). The level shift is incorporated through the variable $LS_{73:II}$, which is 0 before 1973:II and 1 thereafter. Its effect is to lower the estimated value of the quarterly real wage increase by approximately 0.54 percentage point in 1973:II and later. (The change in the level does not equal the coefficient of the $LS_{73:II}$ variable, 0.4011, due to interaction with the autoregressive parameters of the model.)

The forecast for the real wage increase over the next 20 quarters, as generated by the stochastic model, is also shown above in chart 10. For all but the first few quarters, the forecast equals the estimated mean of the data for the period 1973:II through 1993. As noted in prior sections, forecasts from autoregressive models such as this one ultimately "revert to the mean" of the historical data. In this instance, the reversion is quite rapid. Since the relationship between adjoining values of the real wage increase is fairly weak, the model places relatively more weight on the mean of the data and relatively less on the values of the series in recent prior quarters.

In the absence of the level-shift variable, the model forecasts would quickly tend to the overall average level for the real wage increase of about 0.3 percent per quarter. Excluding the level shift, however, would generally reduce the accuracy of the near-term forecasts (see discussion of this issue in appendices A and E).

Also shown in chart 10 are the 90-percent prediction intervals associated with the forecasts for each of the next 20 quarters. For the fourth quarter of 1998, for instance, the real wage increase is estimated to fall within the range of –1.18 to 1.05 percent with a certainty of 90 percent. (The corresponding range on an annual basis is –4.6 to 4.3 percent.) This forecast is subject to the caveats described in the introduction to this study concerning limitations of stochastic models.

In contrast to the results for the other stochastic models in this study, the prediction intervals for the real wage forecast widen only slightly from shorter to longer time horizons. This result occurs because of the weak association in the data noted above. When recent data values provide little guidance as to future values, forecast certainty is reduced and the model places more weight on the mean of the data and on the overall variability exhibited in the past.[22] As usual, the 90-percent probability associated with the intervals shown in chart 10 applies to the expected real wage

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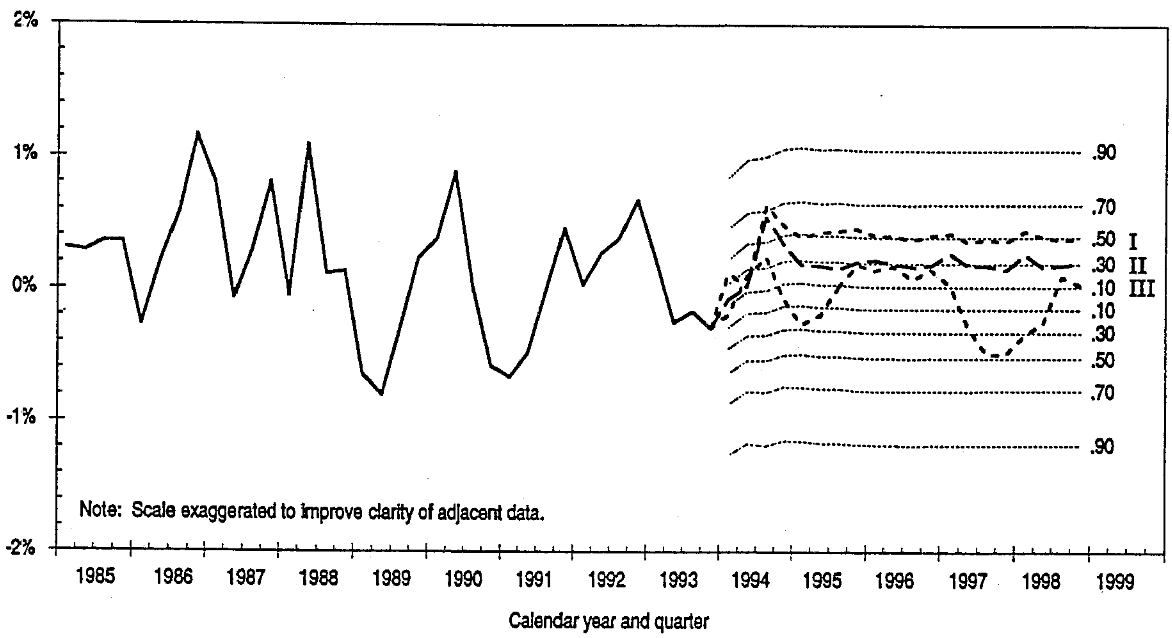
[21] Additive outliers can result from either measurement error or actual phenomena, such as strikes, that temporarily affect the level of the series. The outlier in 1952:IV was also identified by Balke and Fomby [16] using data on manufacturing compensation per hour (not adjusted for inflation). The outlier resulted primarily from large increases in wage levels granted by the Federal government as exceptions to the emergency wage controls then in effect. These controls were subsequently eliminated in February 1953.

[22] Within a few quarters, the 90-percent prediction interval has reached its maximum width of 2.2 percentage points, which is approximately the width of the range containing 90 percent of the original data in the period 1973:II through 1993.

increase for a given quarter and does not represent the probability that all of the real wage increases for the next 20 quarters will fall within the indicated bounds.

Chart 11 compares the assumed quarterly increases in real wages from the 1994 Trustees Report[23] to the series of prediction intervals based on the stochastic model. Except for the first few quarters, the real wage increases assumed under alternative II are very close to the upper bound of the 30-percent interval—significantly above the median forecast from the stochastic model. The alternative I assumptions follow a similar pattern and nearly coincide with the upper bound of the 50-percent interval after 1994.

Chart 11—Comparison of Trustees Report real wage assumptions and forecast intervals based on stochastic model

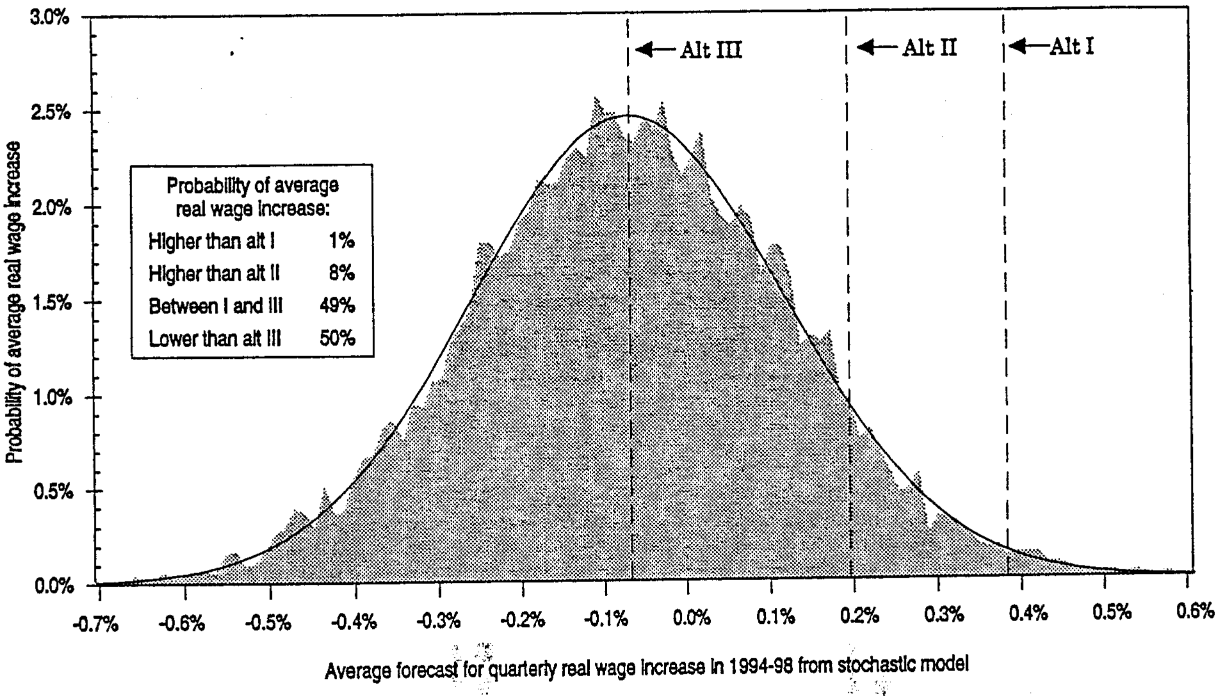


Reflecting the "twin recession" scenario, the alternative III assumptions vary from the lower bound for the 50-percent forecast interval to the upper bound for the 30-percent interval. A portion of this variation is attributable to the underlying inflation rates, which are assumed to increase at the same time that nominal wage increases are restricted by the economic recessions.

Chart 12 compares the average quarterly real wage increase assumed in the Trustees Report for 1994-98 with the distribution of average forecasts from the stochastic model (based on 5,000 individual forecast paths). The distribution of these averages is shown on the following page.

[23] As described in appendix E, these quarterly assumptions were also derived to be consistent with the assumed annual real wage differentials shown in the Trustees Report.

Chart 12—Stochastic probability distribution of average quarterly real wage increase and Trustees Report assumption averages



The 90-percent prediction interval for the average quarterly increase in real wages over the next 5 years is estimated to be –0.38 to 0.25 percent. This range is substantially narrower, as expected, than the corresponding range of –1.18 to 1.05 percent seen previously for the estimated real wage increase in the fourth quarter of 1998. The median of the stochastic distribution is –0.07 percent.

The average quarterly real wage increases assumed for 1994-98 under alternatives I, II, and III are 0.38, 0.20, and –0.07 percent, respectively, as shown superimposed on the distribution in chart 12. The alternative II assumption of 0.20 percent corresponds to the 92nd percentile of the stochastic forecast distribution. In other words, only 8 percent of the stochastic forecasts exceeded the alternative II assumption. Similarly, the probability of an average real wage increase greater than assumed under alternative I was virtually negligible at 1 percent. On the other hand, the alternative III assumption was equal to the median of the stochastic forecasts, with a 50-percent chance of either a higher or lower average forecast.

Overall, these results suggest that there is roughly a 49-percent probability that real wage increases will average between –0.07 and 0.38 percent over the next 20 quarters—the range delineated by the assumptions from the 1994 Trustees Report. This probability is very close to the corresponding figures estimated for the inflation and unemployment rates. The location of the alternative I-to-III range, however, is substantially off-center relative to the stochastic distribution, as indicated above.

The validity of these interpretations, of course, is dependent on the proper specification and estimation of the model and on the assumption that the future will behave similarly to the past. In particular, the location of the stochastic distribution is affected by the level shift variable included in the model. If this level shift were excluded, then the Trustees Report assumption range would be much more centered relative to the model forecasts. (As noted above, however, such exclusion would be expected to worsen the accuracy of the near-term forecasts).

Finally, it must be recalled that this evaluation is based on the quarterly data series and assumptions developed for this purpose and not directly on the annual real wage differentials that are the focus of the Trustees Report. Nonetheless, the quarterly data and assumptions bear a consistent relationship to the corresponding annual figures. Thus, the concerns raised by the analysis of the quarterly data should be equally applicable to the annual real wage differential assumptions. Consequently, serious consideration should be given to the possibility that the current assumptions for future real wage growth over the next 5 years may be too optimistic.

VI. Conclusions

This actuarial study represents the first step in implementing certain of the recommendations of the Technical Panel of actuaries and economists convened by the 1991 Advisory Council on Social Security. The Panel recommended development of methods to help quantify the degree of uncertainty present in the financial projections for the Social Security program and in the key assumptions underlying the projections. They further recommended use of statistical time series techniques as a part of this effort.

In this study, four of the key short-range economic assumptions from the 1994 Trustees Report are evaluated through use of statistical time series models. The primary focus is on the range of possible future variation that is encompassed by the Trustees' low-cost and high-cost assumptions. In addition, consideration is given to the location of the intermediate assumptions within this range. Individual evaluations were performed for the inflation rate, unemployment rate, real interest rate, and real wage increase. In the short range (the next 5 years), these economic factors will have an important influence on the financial operations of the Social Security trust funds.

For each of the economic variables, quarterly data for the last 3 to 4 decades were examined to determine their statistical properties. The correlation between adjoining data values was of particular interest, as were any statistical outliers and level shifts that were identified. Based on the characteristics of the data, a suitable statistical time series model was identified and estimated for each series. A number of tests were applied to determine whether the selected models were properly specified and produced consistent forecasts within the historical period. The final models were used to produce forecasts for the period 1994 through 1998 and probability intervals about those forecasts. The Trustees Report assumptions were compared to these forecast results to evaluate the range of variability encompassed by the assumptions.

Table 5 summarizes the principal results of the evaluation of the Trustees Report assumptions. The probabilities shown apply to the average future value of the economic variable over the period 1994 through 1998. The probabilities are estimated based on the applicable statistical model and are subject to important qualifications (described in section I of this report and reiterated below).

For three of the four series studied—the inflation rate, unemployment rate, and real wage increase—there is roughly a 50-percent probability that the average future value of the series will fall within the range defined by the short-range assumptions in the Trustees Report. The corresponding probability for the real interest rate is significantly lower, at 26 percent. If a uniform probability "width" is considered desirable, then consideration should be given to broadening the alternative I and III real interest rate assumptions.

Table 5—Summary evaluation of Trustees Report assumptions

Probability that average future value of economic variable during 1994 through 1998 will be—	Inflation rate	Unemploy-ment rate	Real interest rate	Real wage increase
More favorable than alternative I	37%	3%	35%	1%
Between alternatives I and II	10%	12%	8%	7%
Between alternatives II and III	35%	38%	18%	42%
More adverse than alternative III	18%	47%	39%	50%
Total	100%	100%	100%	100%
Between alternatives I and III	45%	49%	26%	49%
More favorable than alternative II	47%	15%	43%	8%
More adverse than alternative II	53%	85%	57%	92%
Total	100%	100%	100%	100%

Notes: 1. Probabilities determined based on a sample of 5,000 stochastically generated forecasts from the statistical time series model. See prior sections and appendix A for details.
2. Alternatives I, II, and III are also referred to as the low-cost, intermediate, and high-cost sets of assumptions, respectively.

Generally speaking, the range of variation spanned by the alternative I and II assumptions is significantly narrower than the range defined by alternatives II and III. This result is partly attributable to the nature of the high-cost alternative III assumptions, which are based on an explicit economic scenario incorporating two recessions. In contrast, alternatives I and II represent "trend-average" assumptions and do not explicitly reflect economic cycles. The effect of this difference should be minimal, however, for the average forecasts considered here. Thus, consideration could be given to widening the gap between alternatives I and II, perhaps by making alternative I an explicit, optimistic-cycle scenario in the short range. As such, the low- and high-cost assumptions would represent more equivalent alternatives to the intermediate assumptions. On the other hand, the financial implications of a particularly favorable economic *cycle* versus a steady favorable *trend* are of little consequence, whereas the corresponding difference between *adverse* cycles and trends could have very important implications for trust fund solvency.

Table 5 also indicates that the intermediate inflation rate and real interest rate assumptions are very close to the central or median forecasts from the statistical models. In particular, there is roughly a 50-50 chance of future inflation and real interest rates being either higher or lower than assumed under alternative II. For the unemployment rate and the real wage increase, however, the statistical results suggest that the probability of a result more favorable than alternative II is substantially lower than the probability of a more adverse result. In both cases, in fact, the median or "most likely" forecast from the statistical model coincides approximately with the alternative III assumptions of the Trustees Report. These results warrant further review; if no fault or bias can be found with respect to the underlying statistical models, then careful consideration should be given to the possibility that the unemployment and real wage assumptions are too optimistic in the short range.

As noted in section I of this report, the validity of these statistical forecasts depends critically on whether appropriate models have been identified and accurately estimated—and most importantly on whether the future will continue to behave similarly to the past. The statistical analysis employed in this report offers an additional tool for possible use in the development and evaluation of economic assumptions for the Trustees Report. It is potentially a very powerful tool and is already used widely in economic and financial analysis. Based on standardized, quantitative methods and procedures, it can reduce reliance on individual judgment and opinion. As such, statistical time series analysis offers a potentially valuable addition to the traditional tools used in conjunction with the Trustees Report.

At the same time, the value and importance of individual judgment should not be underestimated. In forecasting the unemployment rate, for example, most analysts would recognize and incorporate an expectation that the current economic recovery will almost certainly lead to further reductions in unemployment for at least another few quarters. The statistical model used in this report, however, does not reflect cyclical trends and merely forecasts an eventual return to an average level of unemployment. The most accurate forecasts may well result from combining the best elements of individual judgment, traditional econometric analysis, and statistical time series analysis.

As a final note, we return to a key question that remains unanswered and consider what steps can be taken to address it in the future. As noted in section II, if the future financial security of the Social Security program rested solely on, say, the inflation rate, then an assumption range spanning only 50 percent of the possible future values would clearly be too narrow. In practice, of course, the trust funds are affected by a number of economic, demographic, and programmatic variables and it is at least somewhat less likely for all of them to simultaneously stray outside the 50-percent range (and in the same direction) than it would be for any one of them alone. Thus, determination of the most appropriate range for the Trustees Report alternatives will require analysis of the statistical interactions among the many factors affecting trust fund financing.

The next stage of research using statistical methods should involve development of a multivariate time series model for the key short-range economic variables studied in this report. Such a model, if feasible, could help determine the extent to which these variables are likely to move in an unfavorable direction simultaneously and should further assist efforts in determining what constitutes the most appropriate range for the alternative assumptions. Ultimately, a multivariate model could also be used to generate interdependent stochastic forecasts for the key economic variables. With estimation of the financial operations of the trust funds for each such forecast, it would be possible to develop a stochastic probability distribution for the level of future assets and to estimate the probability of trust fund exhaustion.

Appendix A—Summary of Statistical Process Used To Evaluate Assumptions

The statistical process used to evaluate the key short-range economic assumptions in the 1994 Trustees Report can be divided into three primary components:

- Data analysis, and model identification and estimation
- Evaluation of model adequacy
- Tests of model's forecasting consistency

In the first stage, the data were analyzed to determine an appropriate stochastic time series model. Models of the autoregressive integrated moving average (ARIMA) variety were considered, due to their flexibility and proven usefulness in analyzing the relationship between current and past values of a data series. Particular attention was paid to the possibility of statistical outliers and level shifts in the data. Based on past patterns of variation in the data, a preliminary model form was identified using the usual identification techniques (described by Box and Jenkins in reference [1]). The parameters of the model were then estimated using either least-squares or maximum-likelihood statistical criteria.

Next, the adequacy of the model was evaluated by examining how well the model fit the data and whether there were any patterns of estimation errors. Conventional measures of goodness-of-fit were calculated and the residuals of the model were reviewed closely for randomness, normality, and constant variance. All tests of significance were performed at the 5-percent level.

Finally, an evaluation was made of the model's forecasting ability over various time horizons up to 5 years in length. If any of the model adequacy and/or forecasting tests raised doubts about the applicability of the model, the process was repeated with appropriate modifications to the model form. These steps are described in more detail below; the results of the process for each of the economic series are summarized in appendices B, C, D, and E for the inflation rate, unemployment rate, real interest rate, and real wage increase models, respectively.

Data analysis and model identification / estimation

The existence of outliers and level shifts in a time series may obscure any underlying structure in the data and may also substantially affect estimates of the parameters of a model (references [8], [25], and [53]). Furthermore, outliers and level shifts are very common in economic time series [16]. Thus, it is important to determine whether outliers are present and what their effect is on any model under consideration. Each of the four data series studied in this report was reviewed carefully for outliers and level shifts using Balke's "combine/reduce" variation of the identification procedure suggested by Tsay (references [14] and [53]). A plot of the data, together with any outliers and level shifts identified, is shown for each data series in the following appendices.[24]

[24] *Level shifts* represent a step-change in the level of the data, either upward or downward, with similar patterns of variation before and after the change in level. *Additive outliers* are characterized by an unusu-

After identification of any significant outliers and/or level shifts, the *autocorrelation function* (ACF) and *partial autocorrelation function* (PACF) were calculated for the data. These functions measure the extent to which data values are correlated to prior data values at progressively earlier points in time.[25] Graphs of both the ACF and PACF are shown for each data series in the appendices. The patterns exhibited by the autocorrelation coefficients can help identify the proper form for a stochastic model to represent the data (see chapters 3 and 6 of Box and Jenkins [1]). A description of the interpretation of ACF and PACF patterns exceeds the scope of this study. It is helpful to recognize, however, that the number of significant PACF coefficients (i.e., those in excess of ± 2 standard deviations) generally corresponds to the order of an autoregressive process. All four of the economic series in this analysis exhibited autoregressive properties and none had significant moving-average characteristics.

Once a plausible model form was identified, the parameters of the model were estimated from the data using either least-squares or maximum-likelihood techniques, depending on the form of the model. Calculations were performed using the *Minitab and SCA Statistical System* commercial software packages, and the *GARCH* Fortran program made available by Dr. Kenneth F. Kroner of the University of Arizona. The resulting model equations are shown in the introductory section of each subsequent appendix.

The final step of this stage of the process involved consideration of the nature and effects of any outliers and level shifts identified in the data. Incorporating known outliers and level shifts into the model specification will often improve the accuracy of near-term forecasts, sometimes dramatically (see references [16], [27], [28], [43], and [53]). Doing so, however, may understate the uncertainty associated with the forecast if additional outliers and level shifts are likely in the future. Thus, there is a trade-off between improving the accuracy of a model's central forecast and accurately measuring the uncertainty associated with that forecast. There are few guidelines available in the statistics and economics literature concerning the optimal selection of model specifications in this circumstance, with the result that the selection of models in this study is somewhat judg-

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ally high or low value in one period only. *Innovational outliers* are similar to additive outliers, except that the effect gradually diminishes over time, following the same pattern of serial correlation exhibited by the underlying data series. Further information on the characteristics and identification of outliers and level shifts is available in Pankratz [8].

[25] For example, the first autocorrelation coefficient, r_1, measures the correlation between each point in the data series and the corresponding value in the prior quarter. Similarly, r_2 measures correlation between the data values and their corresponding values two quarters earlier. In general, the sample autocorrelation function r_k for $k = 1, 2, 3, \ldots$ is given by the following formula, where $\bar{y}$ is the mean of the data series y_t, and N is the number of observations in the series:

$$r_k = \frac{\sum_{t=1}^{N-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{N}(y_t - \bar{y})^2}$$

For $k = 1, 2, 3, \ldots$, the partial autocorrelation coefficient, ϕ_{kk}, is defined as the last coefficient from the kth-order autoregression:

$$y_t = \phi_{k1}y_{t-1} + \phi_{k2}y_{t-2} + \ldots + \phi_{kk}y_{t-k} + \varepsilon_t$$

In effect, ϕ_{kk} measures the additional correlation between data values k periods apart after taking account of the correlation explained at lags 1, 2, 3, ... , k-1 (see Mills [7], chapter 5, for additional detail).

mental. The choices made in the context of this trade-off are discussed in the subsequent appendices for each of the models individually. In general, outliers and level shifts were *not* incorporated whenever possible—that is, whenever their exclusion had no substantive effects on model specification, parameter estimates, or central forecasts.

This decision standard was adopted because of the importance of recognizing that a data series characterized by "unusual" occurrences in the past will likely continue to have new "unusual" occurrences in the future. For example, the oil price shocks of 1973 and 1979 had a major effect on many economic factors, including prices, wages, and interest rates. While another oil shortage may seem unlikely, consider what would have happened if Iraq's 1990 invasion of Kuwait had extended into Saudi Arabia, as it so nearly did.

Thus, for forecasting purposes, one should not necessarily rely on the relatively narrow range of future possibilities that will be predicted from a model that has been vigorously cleansed of past aberrations. In some instances, however, the existence of major outliers or level shifts resulted in a substantial effect on either the structure of the ARIMA model, the estimates of the parameters of the model, or the level of the model's forecasts. In these cases, it was necessary to incorporate the existence of the outliers into the model directly, using the techniques of *intervention analysis* (Box and Tiao [22]). In particular, as will be discussed in the following appendices, the data for the unemployment rate and the real wage increase both exhibited statistically significant and seemingly permanent level shifts in about 1973 and the real interest rate was affected by a major innovational outlier in 1983. It was necessary to incorporate each of these occurrences in the respective models.

Evaluation of model adequacy

The *t ratios* for each model parameter (the ratio of the parameter to its standard error) are shown in parentheses below the parameter in the appendix summaries. Ratios greater than 2 are considered statistically significant at the 5-percent level.

Goodness-of-fit is measured by comparing the standard deviation of the original data to the estimated standard deviation, or *standard error*, of the residuals of the model. A substantial reduction in variation indicates that the model "fits the data" well. The *coefficient of determination* (R^2, adjusted for degrees of freedom) is also used to summarize the percentage of variation "explained" by the model.[26] A comparison of the actual data values versus the "fitted" values is shown graphically in each appendix.

The differences between the actual and fitted values are called the residuals of the model. These are plotted in the appendix together with the results of eight tests to determine if there are

[26] It is important to note that no *causation* is implied in any of the time-series models used in this report. All of the models are based strictly on patterns of variation exhibited by the data over long periods of time. Thus, the word "explain" is used here in the general statistical context of reducing variation through fitting a model that minimizes residual variation. In addition, it is important to note that the R^2 statistic is not always a good indicator of goodness-of-fit when used in time series analysis (reference [7], section 11.8).

extreme values or unusual sequences or runs of data [10].[27] The normality of the residuals can be judged from a *normal probability plot,* which compares the residuals to a corresponding set of randomly generated values from a pure normal distribution. If the paired points fall along a straight line, the residuals are normally distributed. A *correlation test of normality* is also shown as a summary test of normality (Cryer [3]).

Residual values are also plotted against the model's fitted values to investigate whether patterns of errors exist. The resulting plot should be a random "cloud" of data points, without increasing or decreasing trends, or changes in the degree of dispersion.

To further test for independence of the residuals, an autocorrelation function is calculated based on the residuals of the model. If the resulting correlation coefficients are rarely greater than ±2 standard deviations, then there is little danger of autocorrelation. The *Durbin-Watson statistic,* based on the first residual autocorrelation, is also shown and should be close to a value of 2. In addition, the results of the *Modified Box-Pierce test*[28] are shown and should not indicate a lack of fit.

A final test is performed on the residuals to check for constant variance. The *squared* residuals represent a crude measure of the residual variance at each point in time. The variance should be constant over time and at any given point should not exhibit correlation to prior variances; an autocorrelation function is calculated and graphed for the squared residuals as a check for such relations. An overall test for nonconstant variance, or *heteroscedasticity,* is applied based on the Lagrange Multiplier (LM) procedure described by Engle.[29] The residuals for two of the economic series exhibited nonconstant variance. For these series (the inflation rate and real interest rate), it was necessary to employ a generalized, autoregressive, conditional heteroscedastic (GARCH) model to correct the problem (see appendices B and D).

Tests of model's forecasting consistency

In addition to meeting the usual statistical tests applied to time series regression models, summarized above, any model used primarily for forecasting should produce consistent and plausible fore-

[27] Numerous failures among the eight tests are an indication that the residuals are not completely random. It should be noted, however, that a few test failures will be commonly encountered even with completely random data when the sample size is large (as is the case for the economic series evaluated in this report).

[28] This test is based on the sum of squared residual autocorrelations. For the first m lags, the statistic Q given below has a χ^2 distribution with $(m-p-q)$ degrees of freedom, where p is the order of the autoregressive component of an ARIMA model and q is the order of the moving average component.

$$Q = N(N+2)\sum_{k=1}^{m}(N-k)^{-1}\hat{r}_k^2$$

The smaller the values of the residual autocorrelations, the less likely it is that the Q statistic will be significant. For the tests employed in this paper, Q was calculated based on the first 24 residual autocorrelations.

[29] This procedure involves performing autoregressions of successively higher order k on the squared residuals. The statistic $N \cdot R^2$ will have a χ^2 distribution with k degrees of freedom. High values of the statistic suggest a significant relationship among the squared residuals over time, indicating that the variance is unlikely to be constant. See Engle [32] for more details.

casts. If a model predicted a significant possibility of negative unemployment rates, for example, then serious defects in the model specification would have to be considered.

For each of the data series, the fitted model (based on the full data set) was used to calculate forecast prediction intervals for each data point in the set. These intervals were compared to the actual values at each point, and the percentage of actual values lying outside the associated prediction intervals was calculated. Comparisons were made for 1-quarter-ahead forecasts, 4-quarters-ahead (1 year), and 20-quarters-ahead (5 years). The evaluation was based on 90-percent intervals; barring specification or other problems, the forecast intervals should fail to capture the actual experience approximately 10 percent of the time.[30]

The last page of each appendix presents graphs of the actual data values compared to the 1-, 4-, and 20-step ahead forecast intervals. Also shown are the percentages of the actual values falling outside the intervals. It should be noted that these tests are designed only to help identify any potential anomalies with the model's forecasts, rather than to serve as an evaluation of actual past forecasting performance in comparison to other forecasting methods. An evaluation of the latter type would also be of interest and should be considered for further research.

Caveats

As noted elsewhere, these models are intended for the purpose of evaluating the range of variation present in the key short-range economic assumptions used in the 1994 Trustees Report. To the extent that the models faithfully measure and reflect actual past patterns of variation, they should be fully appropriate for such use.

It is important to remember, however, the many limitations of such models. Their estimation is subject to measurement errors in the data and to estimation errors resulting from limited data availability. Moreover, the very important implicit assumption is made that the future will generally behave similarly to the past. This assumption is reasonable for the purpose at hand. In practice, however, experience has demonstrated many times over that the future can be unexpectedly different.

Thus, these models should be considered from the perspective of George E. P. Box's statement that "Models ... are never true, but fortunately it is only necessary that they be useful."[31]

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[30] Random variation can result in a failure proportion that differs from 10 percent. For series of the length studied here, the proportion should not be less than about 6 percent nor greater than about 14 percent.

[31] Reference [23].

Appendix B—Statistical Model for Inflation Rate

As noted in the main body of this report, the inflation rate was calculated using the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W) over the period 1949:I through 1993:IV. The data are not seasonally adjusted. Quarterly values for the CPI-W (which is a monthly series) were set equal to the value for the middle month of each quarter.[32] The inflation rate c_t was defined as the increase in the CPI-W from quarter $(t-1)$ to quarter (t), expressed in percent:

$$c_t = 100\left(\frac{CPI_t}{CPI_{t-1}} - 1\right)$$

Development of a statistical model for the inflation rate began with an analysis of the data as calculated above and shown in the top panel of chart B1. The data were characterized by gradual changes in trend over time (producing a "meandering" appearance) and occasional bursts of volatility. The search for outliers and level shifts turned up three additive outliers and one innovational outlier, all in the early years, and three statistically significant level shifts, in quarters 1967:III, 1973:I, and 1981:IV.

The second and third panels in chart B1 present the autocorrelation and partial autocorrelation functions, respectively, for the inflation rate data. The ACF and PACF were not significantly affected by adjusting the data for the above outliers and level shifts; the functions shown are based on the unadjusted data. The large and persistent autocorrelation coefficients reveal a strong association between current and prior values of the inflation rate. The patterns suggest a third-order autoregressive or AR(3) model, with the first three PACF coefficients in excess of +2 standard deviations. Subsequent analysis, however, was performed using an AR(4) model specification—in part to capture possible seasonal effects but primarily because a significant body of prior research exists based on an AR(4) specification (references [16], [20], and [36]).

An AR(4) model was estimated based on the inflation data, both with and without adjustment for the outliers and level shifts. The autoregressive structure of the model was not affected by inclusion of the additional variables to reflect the outliers and level shifts, nor were the AR parameters significantly affected. Moreover, the current average level of the CPI increase, as measured by the final level shift in the data, was not markedly different from the overall average for the period 1949 through 1993 (0.85 percent versus 0.99 percent, per quarter, respectively). Since the median forecasts from each model will gradually move toward the final or overall mean of the data, respectively, the inclusion or exclusion of the level shift variable has little effect on the median forecasts. Consequently, the outliers and level shifts were not incorporated in the model on the principle that additional outliers and level shifts can be expected to occur on occasion in the future. Removal of these effects in the past could unduly narrow the range of forecasts for the future, particularly over longer time horizons, thereby understating the true degree of uncertainty associated with the forecasts.

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[32] A more common practice is to use the average of the three monthly CPI values for the quarter. In time series analysis, however, aggregating data in this way can lead to spurious correlations [45].

Fitting the standard AR(4) model resulted in residuals that were deficient in several important respects. In particular, they were not normally distributed and did not have constant variance through time. These results suggested use of a nonlinear model allowing for the variance of the error term to change over time. The class of generalized, autoregressive, conditional heteroscedastic (GARCH) models was developed to address such problems (Engle [32] and Bollerslev [20]) and has found wide application in financial and economic modeling (Bollerslev *et al.* [21]). An AR(4)/GARCH(1,1) model was estimated for the inflation rate data and is shown below. The t-statistics for the model parameters are shown in parentheses under the parameter estimates.

$$
\begin{aligned}
c_t = &\ 0.1342 + 0.5409c_{t-1} - 0.0010c_{t-2} + 0.3724c_{t-3} - 0.0474c_{t-4} + \varepsilon_t \\
&\ (1.78) \quad (10.69) \quad (-0.01) \quad (4.03) \quad (-0.67)
\end{aligned}
$$

with $\varepsilon_t \in N(0, \sigma_t^2)$ and

$$
\begin{aligned}
\sigma_t^2 = &\ 0.0250 + 0.1678\varepsilon_{t-1}^2 + 0.7441\sigma_{t-1}^2 \\
&\ (1.35) \quad (1.95) \quad (5.99)
\end{aligned}
$$

This model estimates the current inflation rate as a weighted average of the rates in the prior 4 quarters. The parameter estimates for the second- and fourth-order lags (c_{t-2} and c_{t-4}) are not statistically significant and contribute little to the utility of the model. These parameters were retained, however, for ease of comparison to other research results.[33]

The error term ε_t is normally distributed with mean zero, but the variance σ_t^2 is allowed to change over time. In this way, the alternating periods of "calm" and "volatility" exhibited by the inflation rate data can be incorporated into the model. During periods of relatively stable variation in the inflation rate, such as the early 1960s and the last 3 years, the model errors are small and the variance for the model diminishes over time. This trend leads to forecasts with somewhat narrower prediction intervals. At other times when the inflation rate exhibits more variability, the variance term increases to reflect the increased uncertainty. Forecasts originating from such periods will reflect larger (less certain) prediction intervals.

The bottom panel in chart B1 compares the actual inflation rates to the estimated values based on the statistical model. The standard error of the estimates (0.63) represents a significant improvement from the standard deviation of the original data (0.94), but a substantial degree of variation remains. The R^2 value, adjusted for degrees of freedom, is 54.5 percent. Both measures would be substantially improved through use of a model incorporating the seven outliers and level shifts described earlier. Recall, however, that our goal is to evaluate the range of possible future variation in inflation rates. To the extent that the future will probably also have "unusual" variation due to shortages, wars, etc., then cleansing the past of such effects may unduly narrow our outlook for the future.

---

[33] In other research findings, these parameters are frequently significant at the 5-percent level. This result could be due to the use of seasonally adjusted CPI data in the other studies, as opposed to the seasonally unadjusted data used here, if the adjustment for seasonality is excessive. The t-ratios for c_{t-1} and c_{t-3} are highly significant in all of the other studies, as in this one.

Chart B2 presents the results of a number of tests for model adequacy. The upper panel indicates that the standardized residuals of the model, calculated as ε_t/σ_t, are randomly distributed, with only the occasional randomness test failure that is typical of large-sample random data. A comparison of the standardized residuals to a corresponding set of values drawn from a true normal distribution reveals that the standardized residuals are normally distributed (second panel). A plot of the residuals vs. the fitted values from the model shows no sign of nonrandom patterns or changes in variance (third panel). The bottom two panels indicate that the standardized residuals show no signs of autocorrelation or nonconstant variance. Thus, all of the residual tests are met satisfactorily.

A test of the model's suitability for forecasting is provided by chart B3. In the top panel, the model is used to forecast the inflation rate for the following quarter, based on data up to that quarter. Such forecasts are made starting from each of the 180 data points comprising the original data series (except for the first four, which must be used as "starting values" for c_{t-1}, c_{t-2}, c_{t-3}, and c_{t-4} in the model). For each forecast, an approximate 90-percent prediction interval is calculated.[34] The upper and lower limits of the series of prediction intervals are shown in gray together with the original inflation rate data. The 90-percent intervals should fail to capture the true inflation rate about 10 percent of the time. As indicated, this "outlier percentage" is 11.7 percent for the 1-step-ahead forecasts—a result that is reasonably close to expectations. An unusually high or low outlier percentage would be an indication that the statistical model fails to adequately reflect the patterns of variation exhibited by the data.

The *width* of the prediction intervals for the 1-step-ahead forecasts changes over time, corresponding to whether recent inflation rates have been steady or volatile. The substantial variation seen in the early 1950s, for example, is reflected in very wide prediction intervals. Conversely, during the relative calm of the late 1950s and early 1960s, prediction intervals narrow significantly. Notice that the wider forecast intervals are generally associated with the occurrence of the outliers and level shifts described previously. Thus, the GARCH model reflects the greater uncertainty caused by unusual occurrences in the data by widening prediction intervals. When inflation rates have been fairly stable for several years (as is the case currently), the estimated residual variance is reduced and the forecast intervals for the immediate future are narrowed.

The bottom two panels of chart B3 present similar comparisons based on forecasts for 4 quarters ahead (1 year) and 20 quarters (5 years). In both cases, the percentage of actual values falling outside the approximate 90-percent prediction intervals is reasonably close to 10 percent. Over longer time horizons, the central forecasts tend to converge to the mean of the data and the prediction intervals tend to converge to a width consistent with the dispersion of the original data (Engle and Kraft [36]). In other words, knowledge of the data in the recent past becomes progressively less relevant as the forecast period increases. Eventually, the best forecast (in the statistical sense) reflects only the mean and variance of the full set of data.

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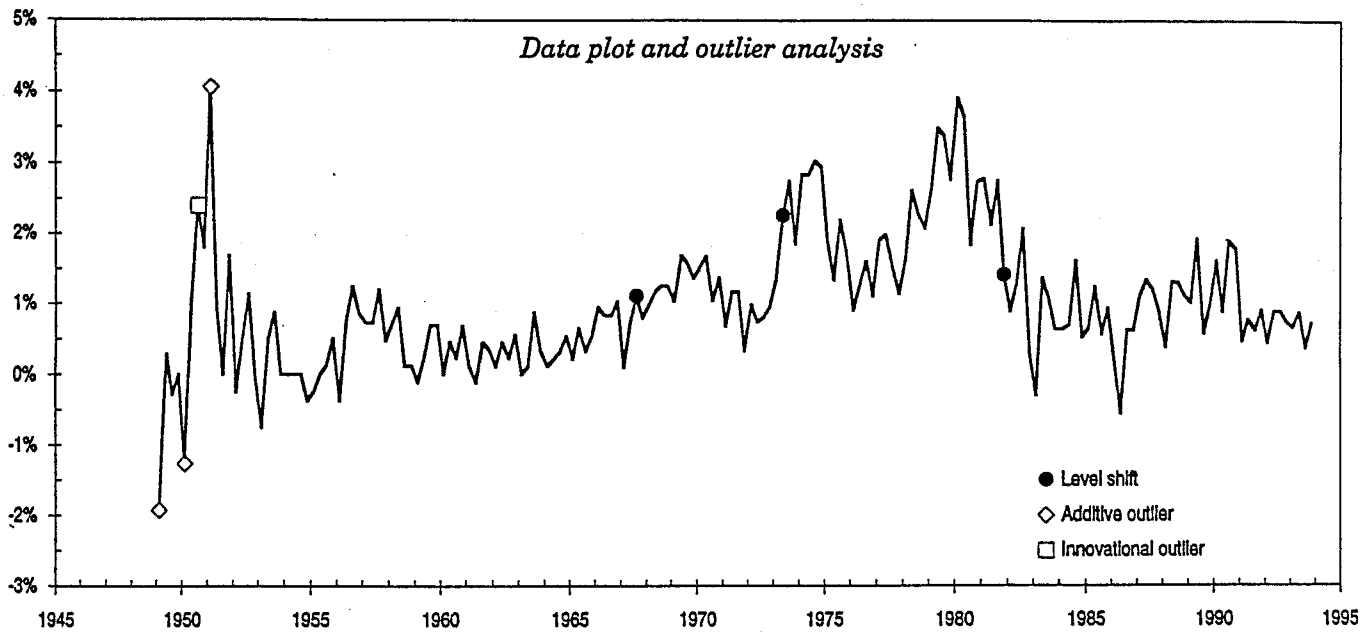
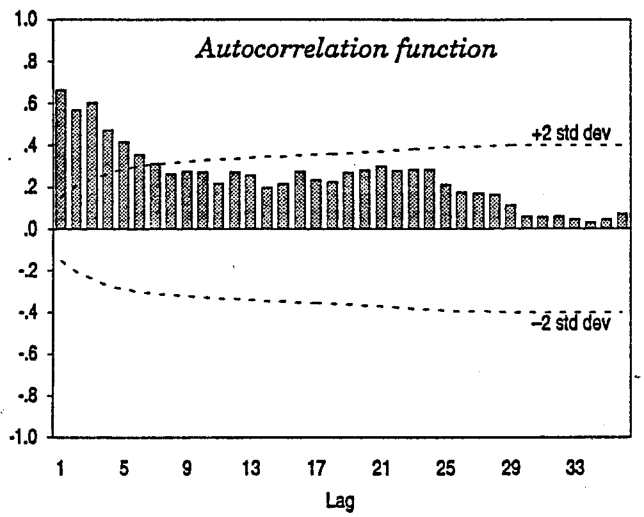
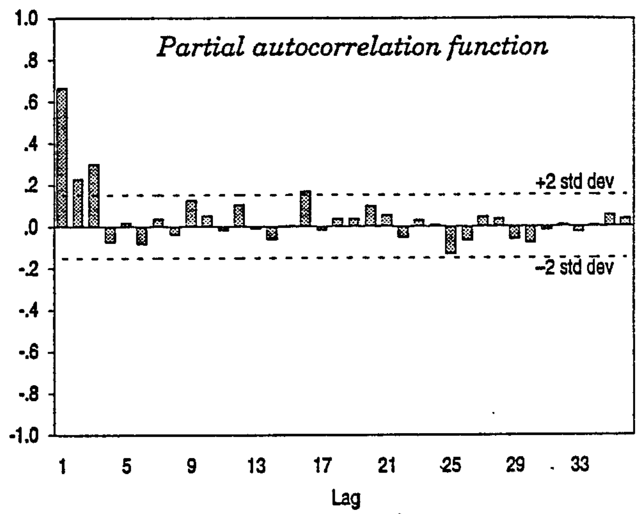
[34] The distribution of forecast errors for GARCH models is non-normal, typically with "fatter tails" than present in a normal distribution (Baillie and Bollerslev [12]). The prediction intervals shown here and in section II for the inflation rate model are approximations, based on the simplifying assumption that the distributions are normal. Monte Carlo tests indicate that this assumption is reasonably accurate in this instance.

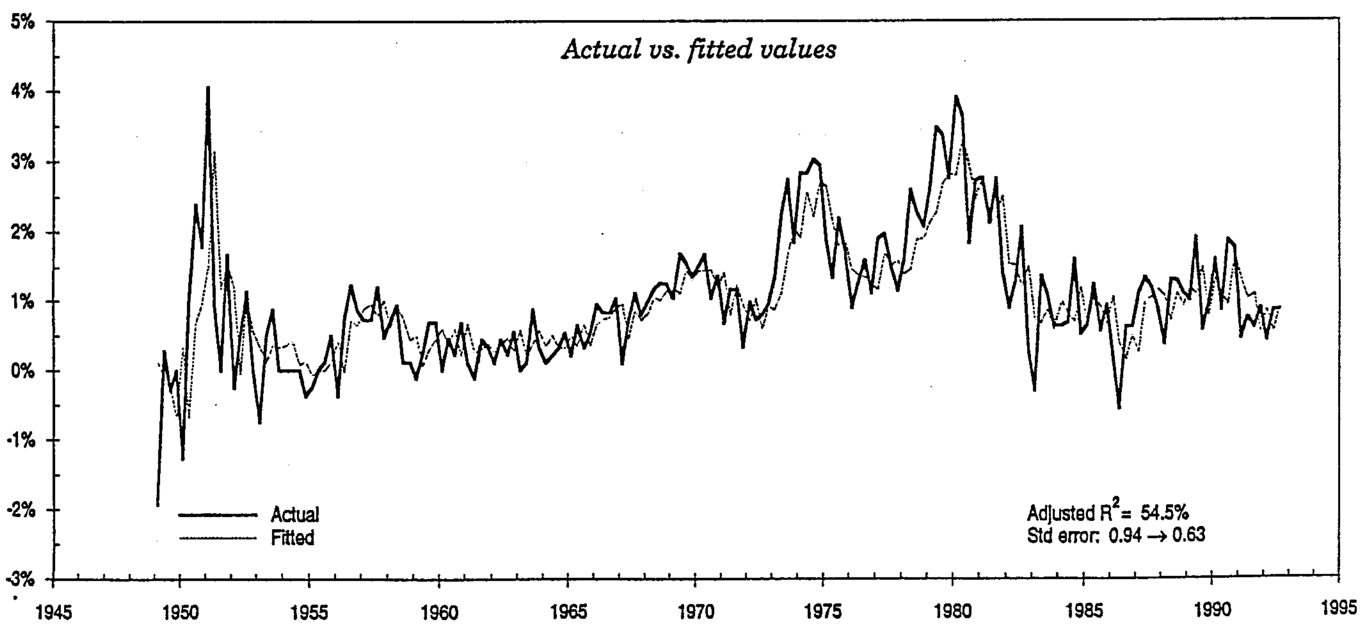
The results of these various tests of model specification, goodness-of-fit, and forecasting consistency suggest that the AR(4)/GARCH(1,1) model for the inflation rate is suitable for the intended use of evaluating the short-range inflation assumptions in the 1994 Trustees Report.

Appendix Chart B1—Identification and fit for inflation rate model

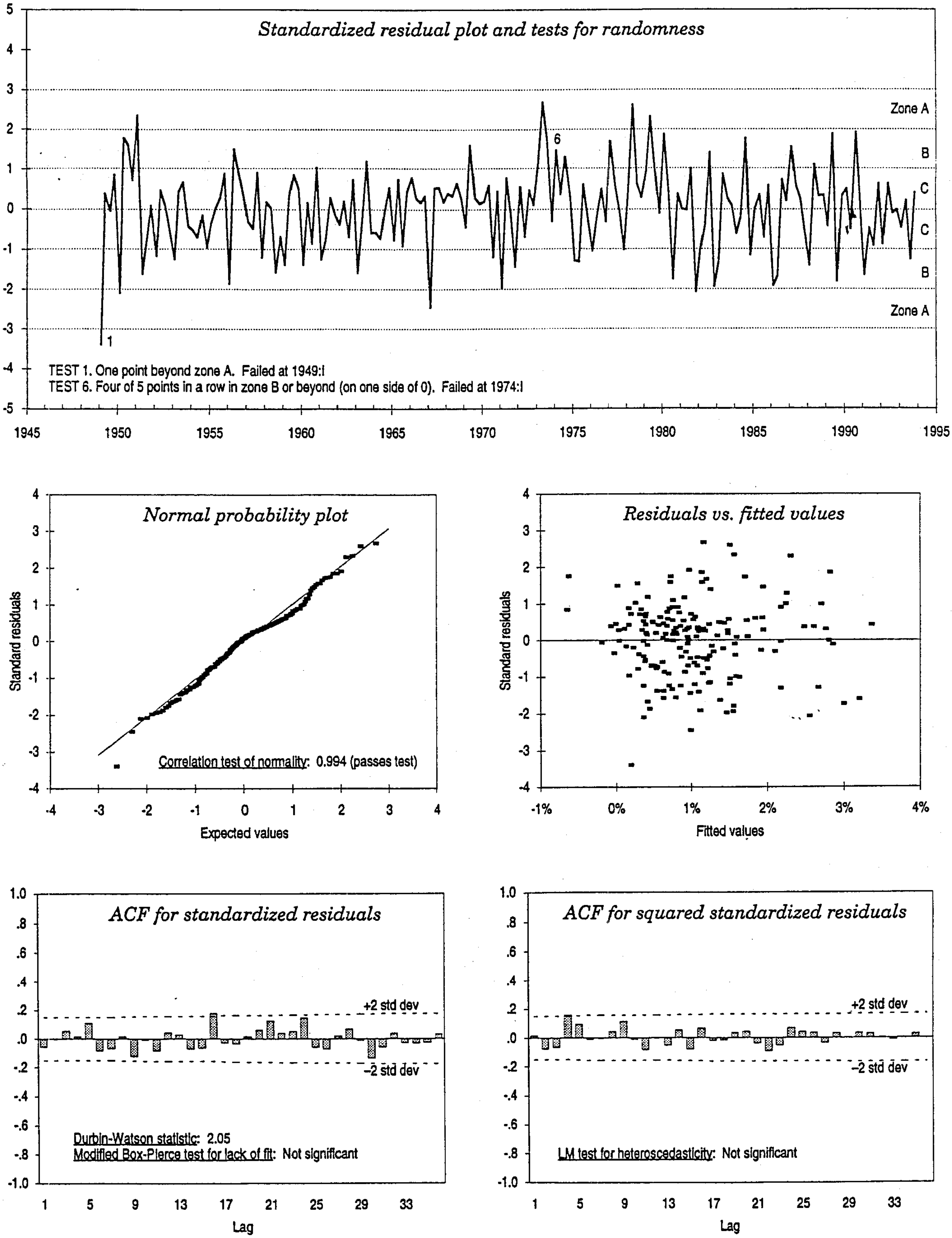




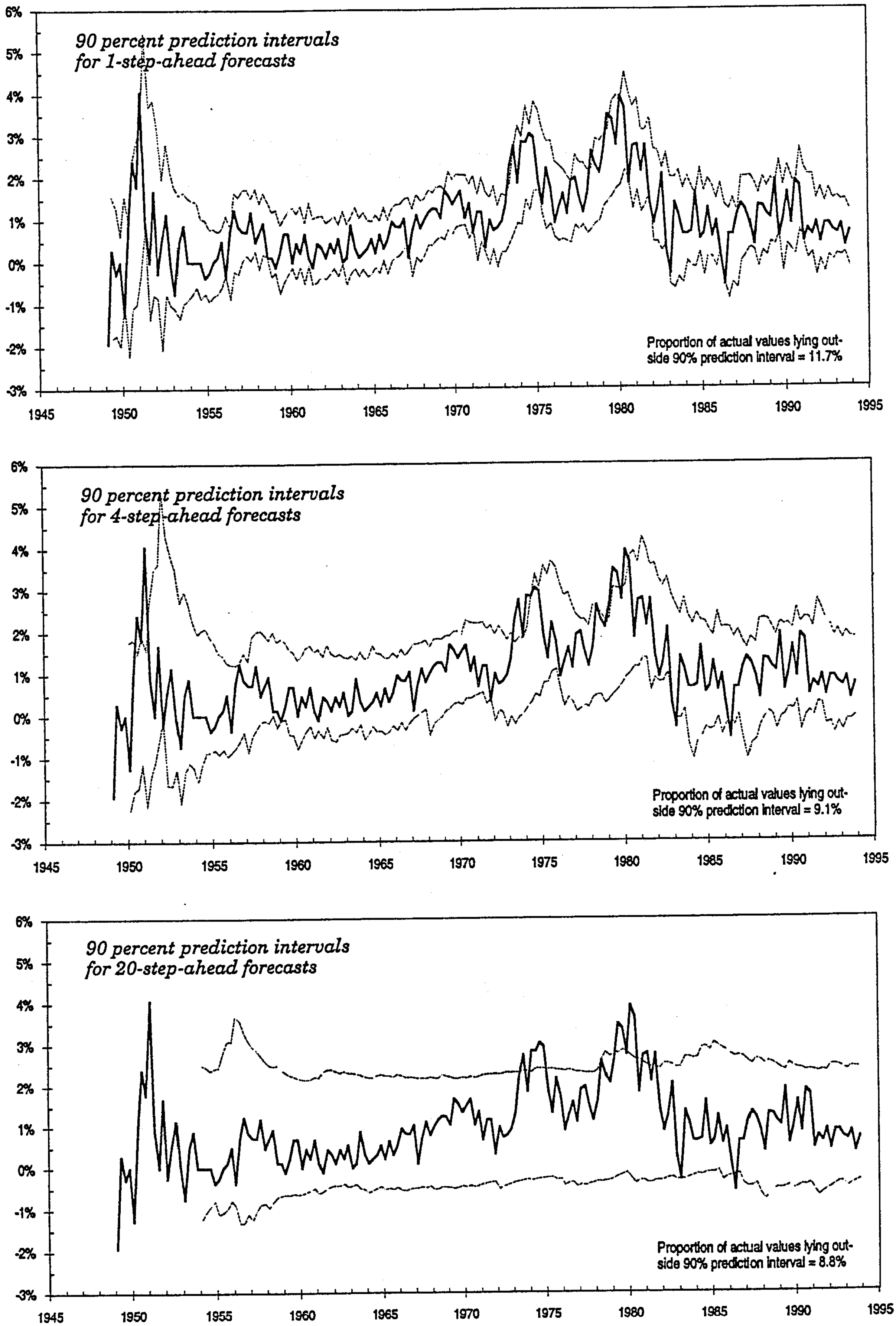




Appendix Chart B2—Residual analysis for inflation rate model



Appendix Chart B3—Within-sample forecast intervals for inflation rate model



Appendix C—Statistical Model for Unemployment Rate

The unemployment rate analysis was based on the seasonally adjusted civilian unemployment rate for the period 1950:I through 1993:IV. The data are based on the Bureau of Labor Statistics' pre-1994 methodology for measuring unemployment and thus do not reflect their latest survey definitions. Quarterly values for the unemployment rate (which is a monthly series) were set equal to the value for the last month in each quarter.[35]

An analysis of the unemployment rate data, as shown in the top panel of chart C1, revealed a cyclical pattern of variation reflective of the business cycle. The search for outliers and level shifts identified several additive and innovational outliers, mostly in the early years, and a major level shift starting in 1974.[36] Superimposed on the data is a plot of the average unemployment rate during 1950 through 1973 and the corresponding average based on data in 1974 and later. For each section of the graph, the unemployment rate appears to vary about the applicable mean in a cyclical fashion.

The autocorrelation and partial autocorrelation functions for the unemployment rate data were not significantly affected by the existence of the additive and innovational outliers. The level shift, however, had a major effect on the correlations. In the absence of an adjustment for the level shift, the ACF showed an extremely high degree of correlation—normally, a sign that the time series is *nonstationary* (for example, does not vary about a fixed mean). Separate ACFs based on data for 1950-73 and 1974-93, however, each revealed a pattern associated with a *stationary* second-order autoregressive or AR(2) model, with the first two partial autocorrelation coefficients in excess of ± 2 standard deviations. The ACF and PACF shown in the second and third panels of chart C1, respectively, are based on the data for 1974-93 only; the functions for the earlier period are very similar.

Several models were estimated based on the unemployment data, with and without adjustment for the outliers and level shifts. To investigate the nonstationary model suggested by the ACF of the data unadjusted for outliers and level shifts, an ARIMA(0,1,2) model[37] was estimated. The results of this exercise were unsatisfactory. In particular, the forecast intervals associated with true nonstationary data series are extremely wide—in this instance, within the sample period there were virtually *no* actual values falling outside the 90-percent forecast interval. Moreover, the

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[35] As noted in appendix B, a more common practice is to use the average of the three monthly unemployment rates for the quarter. In time series analysis, however, aggregating data in this way can lead to spurious correlations [45].

[36] Two distinct level shifts were actually identified, one at 1974:IV and the other at 1980:II, as shown in chart C1. Because the level implied by each is approximately the same, however, they were treated jointly as one shift in the analysis.

[37] This model is based on the *changes* (also called *first differences*) in the unemployment rate from one quarter to the next, rather than on the level of the unemployment rate in each quarter. The model specification is:

$$\nabla y_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$$

where $\nabla y_t = y_t - y_{t-1}$ and ε_t is the difference between the actual and estimated values at time t.

lower bounds for the 20-quarter-ahead prediction intervals were *negative*. Because of these deficiencies, no further consideration was given to nonstationary models for the unemployment rate.

Acknowledgment of the level shift in the data resulted in an AR(2) model, as suggested above. Inclusion of the additive and innovational outliers had no effect on the model specification and relatively little effect on the parameter estimates. Accordingly, only the level shift was incorporated into the model. Inclusion of the level shift is likely to improve the accuracy of the model's near-term forecasts; forecast uncertainty, however, may be understated somewhat to the extent that another level shift could occur in the future. This latter concern is minimized in the short range, however, by the relatively infrequent occurrence of level shifts in the past (only one significant shift in 44 years).

The resulting AR(2) intervention model is shown below. The t-statistics for the model parameters are shown in parentheses under the parameter estimates.

$$\underset{}{u_t} = \underset{(3.99)}{0.4749} + \underset{(21.18)}{1.3936\,u_{t-1}} - \underset{(-7.60)}{0.4931\,u_{t-2}} + \underset{(2.92)}{0.2262\,LS_{74:I}} + \varepsilon_t$$

$$\text{where } LS_{74:I} = \begin{cases} 0 \text{ for } t < 1974{:}\mathrm{I} \\ 1 \text{ for } t \geq 1974{:}\mathrm{I} \end{cases}$$

This model relates the current unemployment rate (u_t) to a weighted average of the rates in the prior 2 quarters (u_{t-1} and u_{t-2}). The level shift in 1974 is incorporated through use of the indicator variable $LS_{74:I}$.[38] Prior to 1974:I, the model estimates the unemployment rate relative to a fixed mean of 4.8 percent. In 1974:I, the model begins a transition to higher unemployment rates, leveling off at 7.0 percent after a few quarters. The nature of this transition will be more apparent below, in the discussion of the model's forecasting performance within the sample period.

In contrast to the statistical model described in appendix B for the inflation rate, the error term for the unemployment rate model (ε_t) is assumed to be normally distributed with mean zero and constant variance. This issue will be discussed further below, in the analysis of the model's residuals.

A comparison of the actual unemployment rates with the fitted values based on the statistical model is shown in the bottom panel of chart C1. The standard error of the fitted values (0.37) is substantially lower than the standard deviation of the original data (1.65). The R^2 value, adjusted for degrees of freedom, is 94.9 percent. Both measures would be improved slightly through use of a model incorporating the additive and innovational outliers described earlier. Since the outliers largely reflected normal business cycle variation, however, their incorporation into the model could have led to unduly narrow forecast intervals—that is, prediction intervals that no longer reflected the normal peaks and troughs associated with periods of recession or recovery. For this reason,

[38] The specification for this model, as shown above, differs slightly from the traditional form of level-shift intervention models [22]. Due to the limitations of available computer software, a *gradual level shift* model was estimated (see Pankratz [8], page 320). In models of this type, the level shift occurs gradually over several quarters, rather than all at once in a given quarter.

and because the model's specification and parameter estimates were not significantly affected, the outliers were not incorporated into the model.

Chart C2 presents the results of the tests for model adequacy. The upper panel plots the standardized residuals of the model and shows the results of the eight tests of independence and randomness. As indicated, there are a number of test failures. In particular, the residuals show alternating periods of calm and volatility. As a result, the residuals are not normally distributed, having both too many extreme values and too many that are "not extreme enough." A comparison of the standardized residuals to a corresponding set of values drawn from a true normal distribution reveals that the standardized residuals fail the correlation test of normality (second panel). A plot of the residuals versus the fitted values from the model shows little evidence of nonrandom patterns or changes in variance, however (third panel).

The bottom left panel of chart C2 indicates that the residuals show no significant sign of autocorrelation. The tests for nonconstant variance summarized in the bottom right panel show borderline (but not statistically significant) signs of heteroscedasticity. An AR(2)/GARCH(1,1) model was estimated to investigate this possibility (see appendix B for a description of such models). The residuals of this model also failed the normality and randomness tests, however, and the model provided no improvement in any other respect; consequently, it was not investigated further.

Overall, the model fits the data very closely and the residuals meet some—but not all—of the standard tests. In addition, as will be noted below, the model performed very well in the tests of within-sample forecasting. After much consideration and testing of other specifications, it was concluded that only a nonlinear model reflecting the cyclical nature of the unemployment rate would be able to resolve the remaining concerns. Although a considerable body of research exists for such models,[39] they are beyond the scope of this investigation and will be considered further in future research.

The model's suitability for forecasting is evaluated in chart C3. In the top panel, the model is used to forecast the unemployment rate for the following quarter, based on data up to that quarter. Such forecasts are made starting from each of the 176 data points comprising the original data series (except for the first two, which are used as starting values for u_{t-1} and u_{t-2} in the model). For each forecast, a 90-percent prediction interval is calculated. The upper and lower limits of the series of prediction intervals are shown in gray together with the original unemployment rate data. The 90-percent intervals should fail to capture the true unemployment rate about 10 percent of the time. As indicated, this "outlier percentage" is 10.3 percent for the 1-step-ahead forecasts—a result that is very close to expectations.

The bottom two panels of chart C3 present similar comparisons based on forecasts for 4 and 20 quarters ahead. In both cases, the percentage of actual values falling outside the 90-percent prediction intervals is reasonably close to 10 percent. As seen with the inflation rate model, over longer time horizons the central forecasts tend to converge to the applicable mean of the data and the prediction intervals tend to converge to a width consistent with the distribution of the original

[39] See, for example, the discussion of bilinear and threshold-autoregressive models in Priestly [9] and Markov regime-switching models in Hamilton [40].

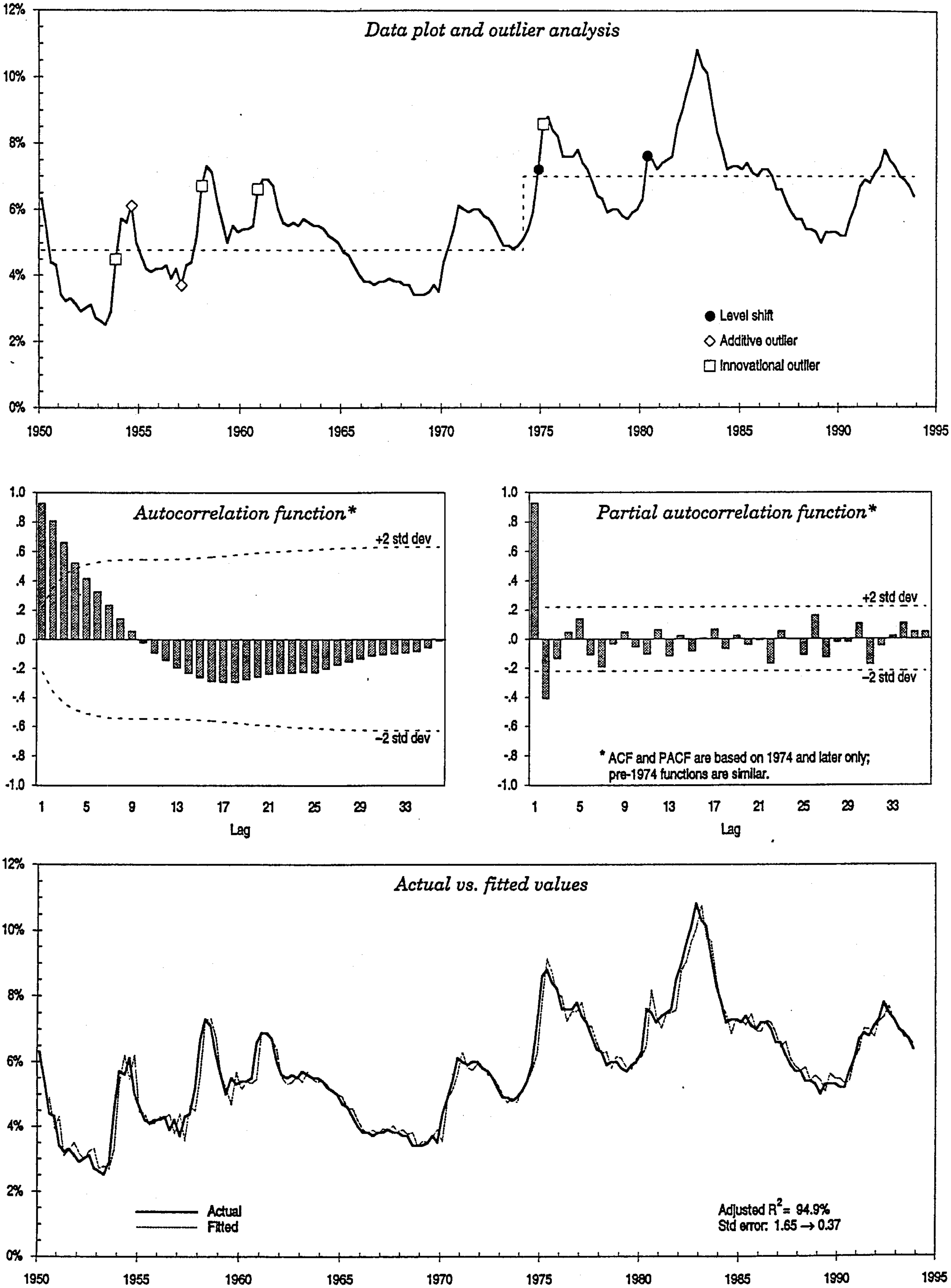
data (after adjustment for the level shift in 1974). As is the case for all of the statistical models considered in this study, knowledge of the data in the recent past becomes progressively less relevant as the forecast period increases.

The last panel of chart C3 also illustrates the nature of the level shift in 1974 and the effect of its incorporation into the statistical model. Starting in the first quarter of 1974, the central forecast (not shown) begins to reflect the transition to the higher unemployment rate. Similarly, the 90-percent prediction interval around the central forecast also gradually rises to reflect the level shift. Both before and after 1974, the actual unemployment rate "wanders" about its mean, occasionally drifting outside the 90-percent intervals on either the high or low sides.

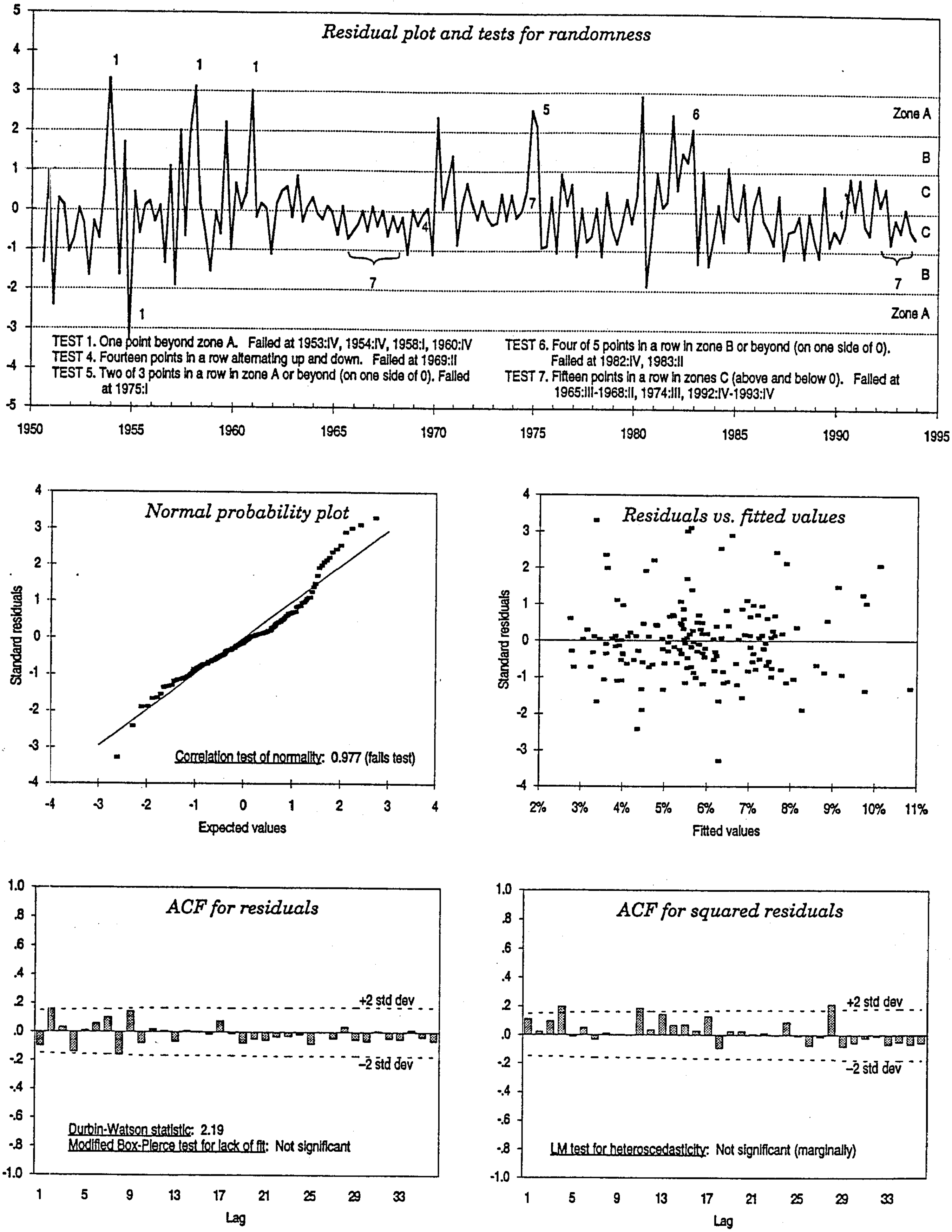
It is important to note that these "within-sample" forecast intervals are used primarily as a crude test for whether the model produces consistent forecasts. For this purpose, the final model is used, based on the full data set and reflecting any outliers incorporated. In practice, of course, forecasts made in an earlier year could only reflect information available up to that time and could not reflect the effects of later data or subsequent outliers or level shifts until they had occurred and had been identified.

The results of these various tests of model specification, goodness-of-fit, and forecasting consistency suggest that the AR(2) intervention model for the unemployment rate is acceptable for the intended use of evaluating the short-range unemployment assumptions in the 1994 Trustees Report.

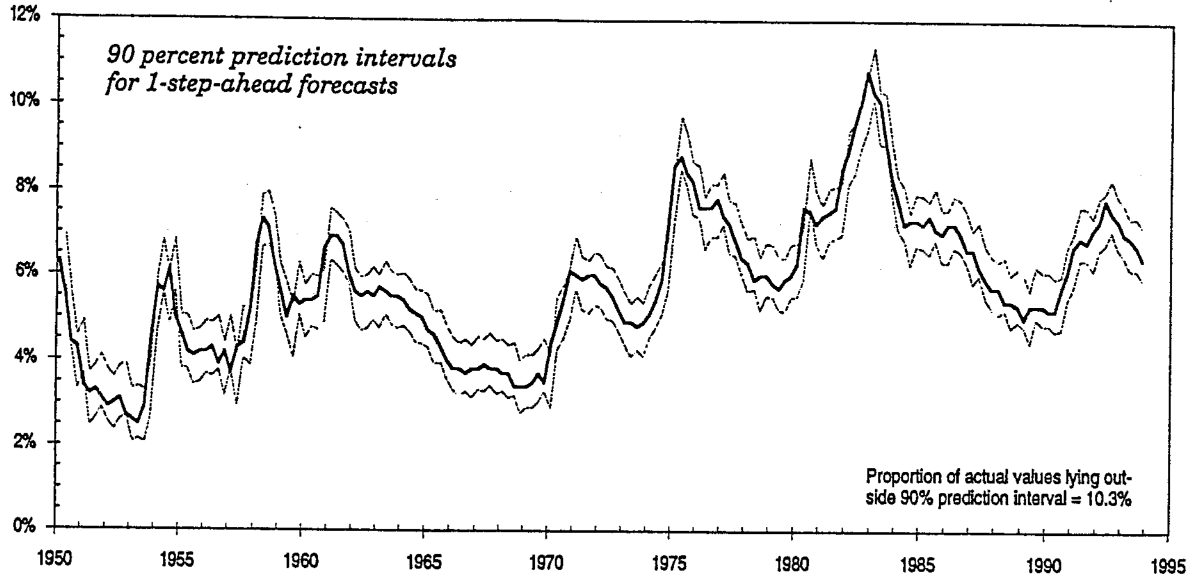
Appendix Chart C1—Identification and fit for unemployment rate model

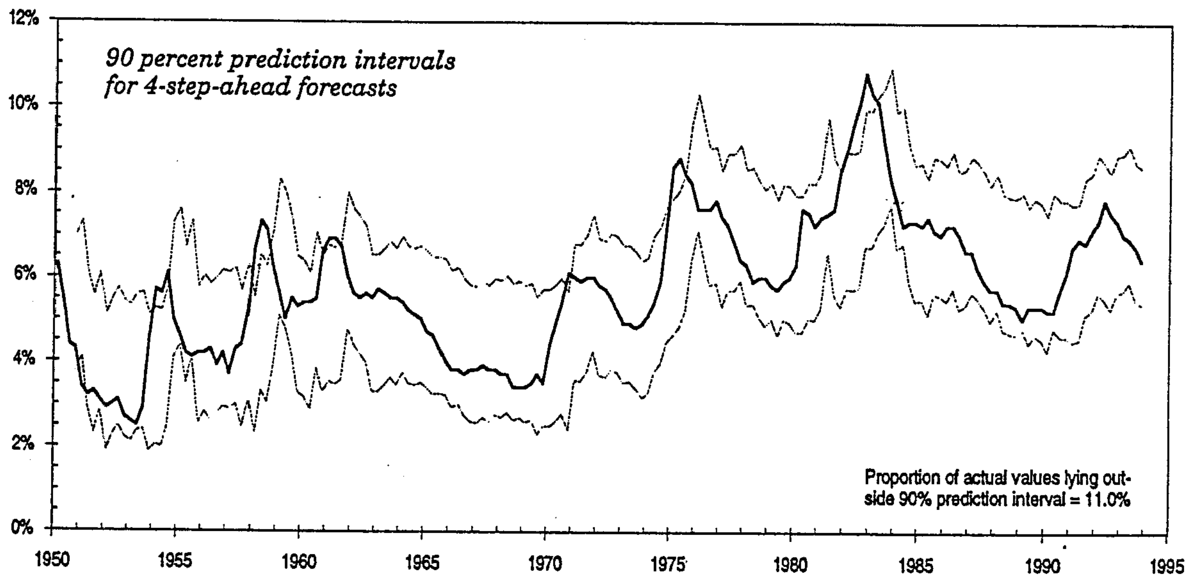


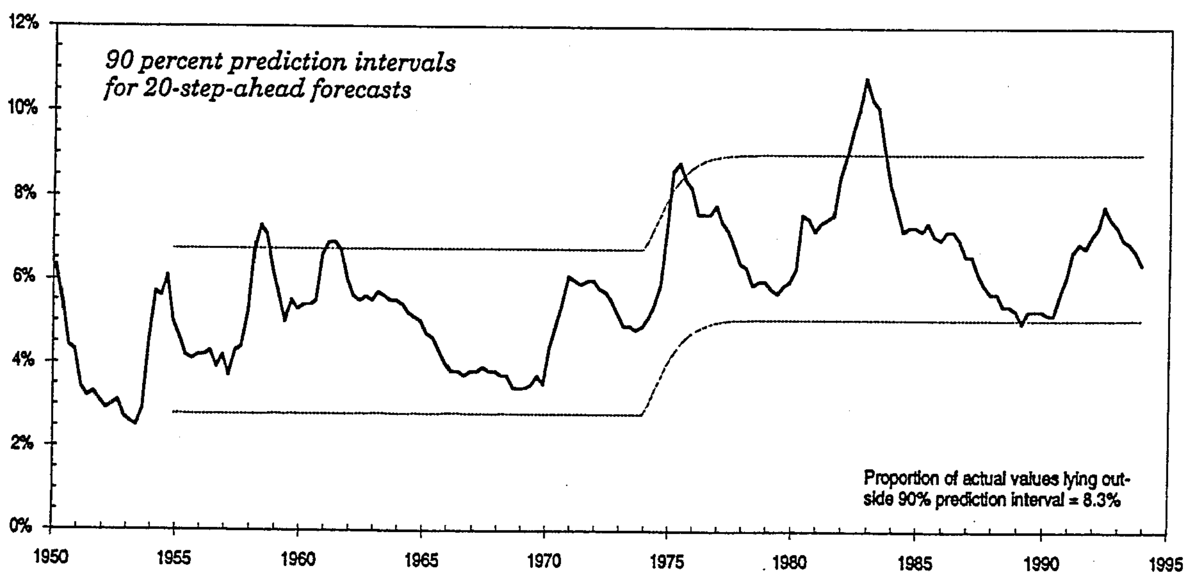
Appendix Chart C2—Residual analysis for unemployment rate model



Appendix Chart C3—Within-sample forecast intervals for unemployment rate model







Appendix D—Statistical Model for Real Interest Rate

The analysis of real interest rates for this study was based on the interest rate payable on new Treasury securities issued to the OASI and DI Trust Funds. By law, the securities issued in a given month carry interest rates equal to the average yield, as of the last day of the prior month, on all outstanding Federal securities that are not due to mature for at least 4 years. Although these "special-issue" securities purchased by the Social Security trust funds carry maturity dates of up to 15 years, the interest rate formula applies regardless of the specific maturity date. Thus, all securities issued in a given month carry the same interest rate. The interest rate data were analyzed for the period 1961 through 1993; trust fund interest rates prior to October 1960 were determined under different investment policies and thus were not relevant to the analysis. Quarterly values for the special-issue interest rate were set equal to the value for the middle month of each quarter.[40]

The nominal interest rates described above were converted to real interest rates using the Consumer Price Index for Urban Wage Earners and Clerical Workers (CPI-W), described in appendix B. To avoid undue influence from the quarterly variation in the CPI, the increase in inflation over the prior year was used.[41] The real interest rate ri_t was defined as the nominal special-issue interest rate i_t in quarter t, adjusted for the increase in inflation from quarter $(t\text{-}4)$ to quarter t, and expressed in percent:

$$ri_t = 100\left(\frac{1+i_t}{\dfrac{CPI_t}{CPI_{t-4}}} - 1\right)$$

The resulting real interest rates are shown in the top panel of chart D1. With the exception of three unusual periods, the data tend to lie in the range of 2 to 4 percent. In the mid-1970s and again in the late 1970s, rapid increases in inflation resulted in large, negative real interest rates. Following the deflationary recession in the early 1980s, the real interest rate climbed dramatically and then began a gradual decline back toward the 2-4 percent range.[42]

The search for outliers and level shifts revealed three that were statistically significant: a level shift in 1981:II, a major innovational outlier in 1983:III, and an additive outlier in 1990:II. Interestingly, the outlier search did not identify any outliers or level shifts associated with the two

[40] As noted in appendices B and C, determining quarterly values as the average of the three monthly values can lead to spurious correlations.

[41] Because the special-issue interest rates are expressed as *annual* rates (e.g., 5 7/8 percent for December 1993), the adjustment by an annual rate of inflation preserves the proper relationship of the units in the numerator and denominator of the formula. Thus, the resulting real interest rates are also expressed as annual rates.

[42] As noted in section IV, other factors may have contributed to the increase in real interest rates, including (i) changes in Federal Reserve policy, (ii) rapid growth in Federal budget deficits, (iii) reduced savings rates in the private sector, and (iv) deregulation of banks and other financial institutions.

periods of large, negative real interest rates. The results of the outlier search are shown in the top panel of chart D1.

The second and third panels in chart D1 present the autocorrelation and partial autocorrelation functions, respectively, for the real interest rate data. The ACF and PACF were not significantly affected by adjusting the data for the above outliers and level shifts; the functions shown are based on the unadjusted data. As seen with the previous models, the large and persistent autocorrelation coefficients indicate a strong association between current and prior values of the real interest rate. The patterns suggest a fourth-order autoregressive or AR(4) model.[43]

An AR(4) model was estimated based on the real interest data, both with and without incorporation of the outliers and level shifts. The autoregressive structure of the model was not affected by inclusion of the additional variables, nor were the autoregressive parameters significantly affected. Careful review of the nature of the outliers and level shifts, in comparison to the characteristics of the data, suggested that the level shift in 1981:II was more of a reaction to the immediately preceding period of very negative real interest rates. As such, it primarily reflected a return to the "normal" level of the data rather than a change to a different, ongoing level.[44] Accordingly, the level shift variable was excluded from the final statistical model. *If* the level shift in 1981:II is valid, and does not represent merely a return to normal levels, then its exclusion from the model may lead to a lower forecast level than is warranted. This question should be resolved with the availability of additional data over the next few years.

The additive outlier in 1990:II had little impact and was excluded from the model. In contrast, the innovational outlier in 1983:III corresponded closely to the abrupt increase in real interest rates and their subsequent gradual decline to more normal levels over the period 1983-88. The nature of the innovational outlier is illustrated by the dashed line in the top panel of chart D1. This curve represents the estimated real interest rate for each quarter taking *only* the innovational outlier into account (and not the autoregressive structure of the data). Prior to 1983:III, this curve follows the average value of the real interest rate. In 1983:III, the estimated value increases sharply; after continuing to increase for another 3 quarters, the estimates decline gradually toward the prior average. After due consideration of its merits and demerits, the innovational outlier was included in the final statistical model. Inclusion of this outlier is believed to improve the accuracy of the model's central forecast in the short range to a small degree. Forecast uncertainty may be understated somewhat to the extent that another period of unusually high real interest rates could occur again.[45]

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[43] The partial autocorrelation coefficients at lags 1, 3, 4, 6, and 23 were statistically significant at the 5-percent level. The significance of four of the first six coefficients suggests the possibility of an AR(6) model, but this specification was not investigated because the more parsimonious AR(4) specification proved satisfactory.

[44] Perron [50] identified a similar level shift using real interest rates for 90-day Treasury bills. Because his data series ended with 1986:III, however, his analysis did not reflect the innovational outlier in 1983:III, which may have tended to overstate the significance of the apparent level shift.

[45] This selection is admittedly somewhat subjective. In practice, however, inclusion of the innovational outlier has relatively little effect on the resulting fit of the model, residual statistics, or forecasting properties. The primary effect is to place somewhat less weight on the period affected by the unusual confluence of factors described in footnote 42.

As was the case for the inflation rate model, the residuals of the statistical model for the real interest rate were not normally distributed and did not have constant variance through time. Thus, the model was modified to incorporate a nonconstant variance for the error term through use of a GARCH(1,1) specification. As noted in appendix B, GARCH models were developed to address such problems and have found wide use in financial and economic modeling. The resulting AR(4)/GARCH(1,1) model estimated for the real interest rate data is shown below, with the t-statistics for the model parameters shown in parentheses under the parameter estimates.

$$\underset{(1.90)}{ri_t = 0.1731} + \underset{(24.19)}{1.0765 ri_{t-1}} - \underset{(-3.44)}{0.1534 ri_{t-4}} + \underset{(3.83)}{2.9072 IO_{83:III}} + \varepsilon_t$$

$$\text{where } IO_{83:III} = \begin{cases} 0 \text{ for } t \neq 1983\text{:III} \\ 1 \text{ for } t = 1983\text{:III} \end{cases}$$

and where $\varepsilon_t \in N(0, \sigma_t^2)$ with

$$\underset{(1.49)}{\sigma_t^2 = 0.0657} + \underset{(1.92)}{0.1967 \varepsilon_{t-1}^2} + \underset{(4.57)}{0.6889 \sigma_{t-1}^2}$$

This model estimates the current real interest rate as a function of the rates in the prior and fourth-prior quarters. (The parameter estimates for the second- and third-order lags were not statistically significant and were deleted from the specification.) The error term ε_t is normally distributed with mean zero and variance σ_t^2, which is allowed to change over time. As with the inflation rate model, use of the GARCH specification reflects the alternating periods of "calm" and "volatility" exhibited by the real interest rate data.

The innovational outlier at 1983:III is incorporated through use of the indicator variable $IO_{83:III}$, which takes the value 1 at t = 1983:III and is 0 elsewhere. The initial impact of this factor is to increase the estimated real interest rate for the third quarter of 1983 by about 2.9 percentage points. Subsequently, the effect varies as a function of the autoregressive parameters ϕ_1 and ϕ_4 in the model—the initial effect increases somewhat at first and then declines gradually over several years, following the same autocorrelation structure established by the model.

The bottom panel in chart D1 compares the past real interest rates to the values estimated by the statistical model. The standard error of the estimates (0.75 percent) represents a substantial improvement from the standard deviation of the original data (2.82 percent). The R^2 value, adjusted for degrees of freedom, is 93 percent.

Chart D2 presents the results of the various tests for model adequacy. The upper panel indicates that the standardized residuals of the model, calculated as ε_t/σ_t, are randomly distributed, with only one test failure (associated with the additive outlier at 1990:II that was not incorporated into the model). A comparison of the standardized residuals to a corresponding set of values drawn from a true normal distribution reveals that the residuals are normally distributed (second panel).

A plot of the residuals vs. the fitted values from the model shows no sign of nonrandom patterns or changes in variance (third panel). The bottom two panels indicate that the standardized residuals are not autocorrelated and have constant variance. Thus, all of the residual tests are met satisfactorily.

The model's suitability for forecasting is evaluated in chart D3. In the top panel, the model is used to forecast the real interest rate for the following quarter, based on data up to that quarter. Such forecasts are made starting from each of the 132 data points comprising the original data series (except for the first 4, which are needed to start up the autoregressive calculation). For each forecast, an approximate 90-percent prediction interval is calculated and shown by the gray curves.[46] As indicated, the prediction intervals fail to capture the true interest rate 9.4 percent of the time for the 1-step-ahead forecasts. Similarly, the "outlier percentages" for the 4-step-ahead and 20-step-ahead forecasts shown in the second and third panels are 9.6 percent and 11.0 percent, respectively. In each case, the result is very close to the theoretically perfect score of 10 percent.

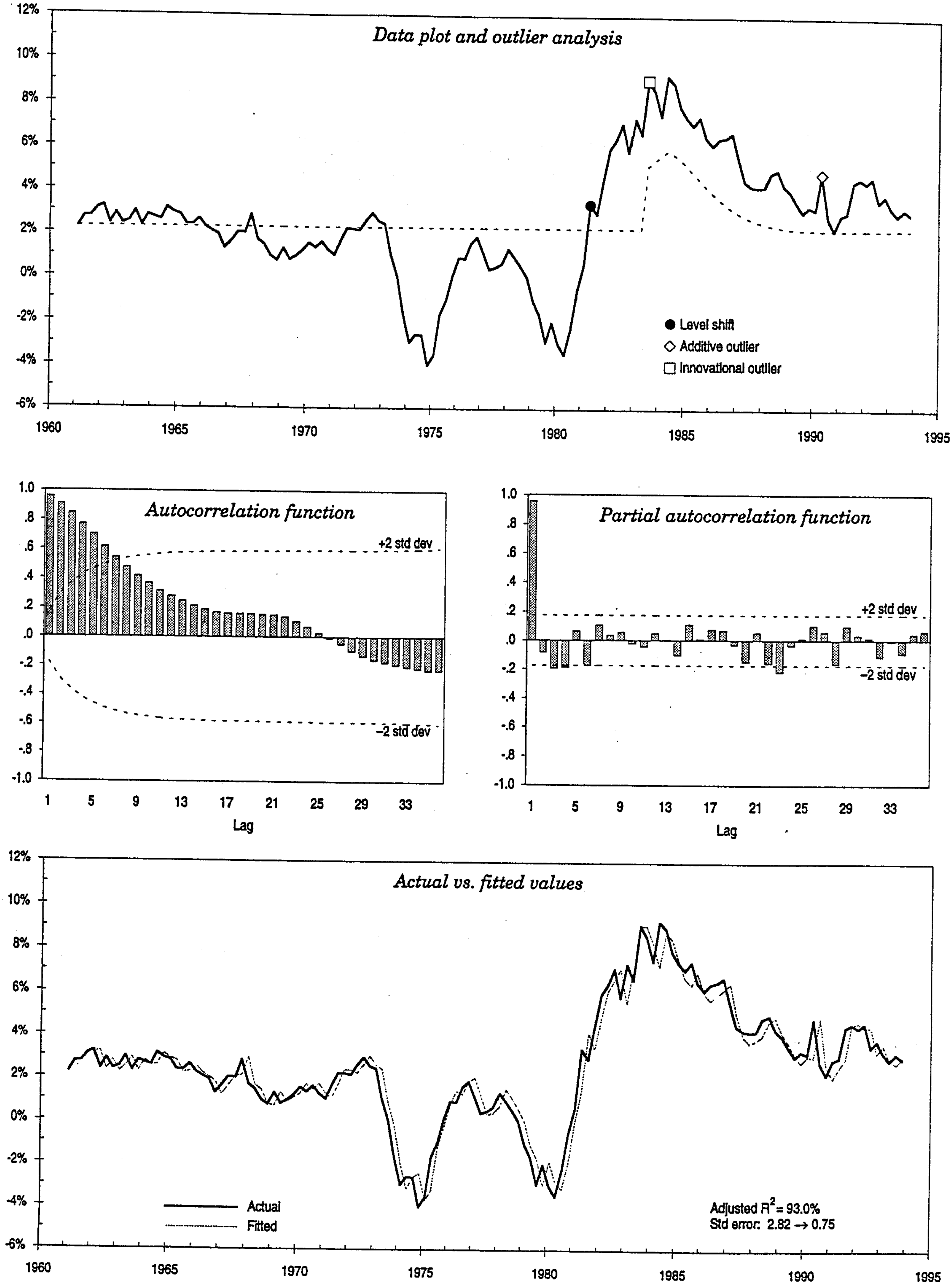
As noted in appendix B for the inflation rate, the width of the prediction intervals for the shorter forecasts changes over time, corresponding to whether recent real interest rates have been steady or volatile. At the widest extreme for the 1-step-ahead forecast, for example, the width of the 90-percent interval is nearly 4 percentage points (1981:III). At the opposite extreme, the 1-step-ahead interval is only 1.6 percentage points wide in 1966:IV. At the end of 1993, the estimated variance of the error term is significantly lower than the average value over the whole period 1961-93.

As noted in appendix C, these tests of forecasting consistency are applied as if the final model, based on the entire data set including the innovational outlier, were available at each point within the sample period. In practice, of course, actual forecasting can be performed based only on the information available to date.

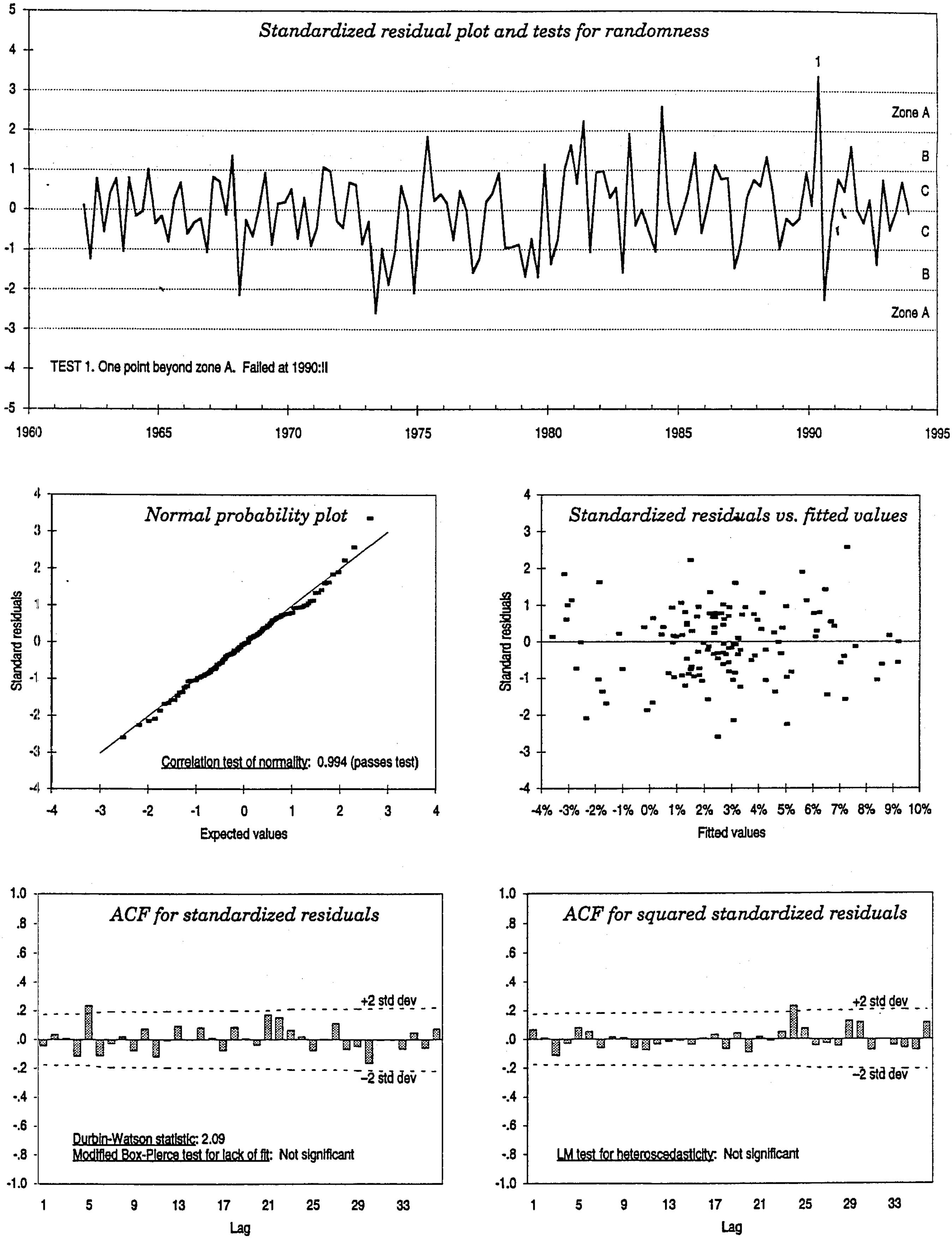
The results of these various tests of model specification, goodness-of-fit, and forecasting consistency suggest that the AR(4)/GARCH(1,1) model for the real interest rate is suitable for the intended use of evaluating the short-range real interest assumptions in the 1994 Trustees Report.

[46] As noted in appendix B, the forecast errors for GARCH models are not normally distributed. Monte Carlo tests indicated that forecast intervals for the real interest rate, approximated by assuming a normal distribution, were very close to the actual intervals. Such approximations are used here for ease of computation.

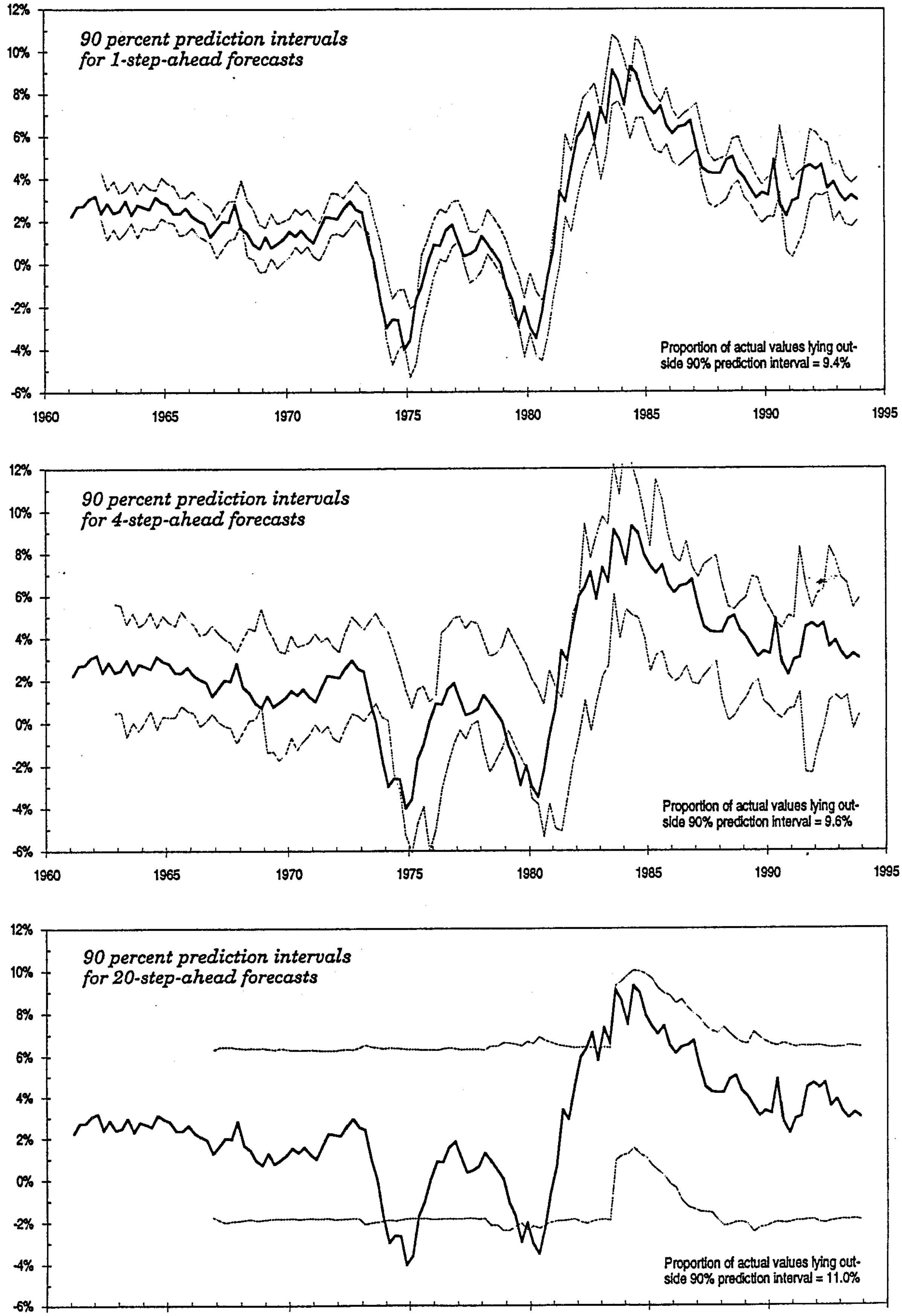
Appendix Chart D1—Identification and fit for real interest rate model



Appendix Chart D2—Residual analysis for real interest rate model



Appendix Chart D3—Within-sample forecast intervals for real interest rate model



Appendix E—Statistical Model for Real Wage Increase

As noted in section V, real wage assumptions for the annual Trustees Report are developed on the basis of annual data. For the stochastic evaluation of the Trustees Report assumptions, however, use of a *quarterly* data series was desired. There were three reasons for this preference. First, a quarterly statistical model is consistent with and facilitates comparison with the models developed for the other economic series under study. Second, as noted in section VI, the next avenue for further research will require the development of multivariate models with recognition of the statistical interactions among the different economic time series. Such analysis will require a common time period and interval for each of the data series. And third, as a practical matter, use of annual data would have limited the data set to only 42 historical values—generally considered too few for statistical time series analysis using ARIMA models (reference [8], page 23).

Accordingly, it was necessary to develop a quarterly data series consistent with the annual data on the real wage differential[47] underlying the Trustees Report projections. The series of *nominal* average wages by quarter was formed using:

(i) aggregate wage and salary accruals (seasonally adjusted) from the National Income and Product Accounts as estimated by the Bureau of Economic Analysis, Department of Labor;[48] divided by

(ii) total wage and salary workers (seasonally adjusted) from the Household Survey of the Bureau of Labor Statistics, Department of Labor.

Based on the nominal average wage amounts described above, the quarterly real increase in average wages was calculated using the CPI-W to adjust for inflation. To avoid undue influence from the quarterly variation in the CPI-W, the average quarterly increase in inflation over the prior year was used. Thus, the real increase in the average wage in quarter t, designated rw_t, was defined as the increase in the nominal average wage, AW_t, adjusted for the average quarterly increase in inflation from quarter $(t\text{-}4)$ to quarter t, and expressed in percent:

$$rw_t = 100\left[\frac{\dfrac{AW_t}{AW_{t-1}}}{\left(\dfrac{CPI_t}{CPI_{t-4}}\right)^{\frac{1}{4}}} - 1\right]$$

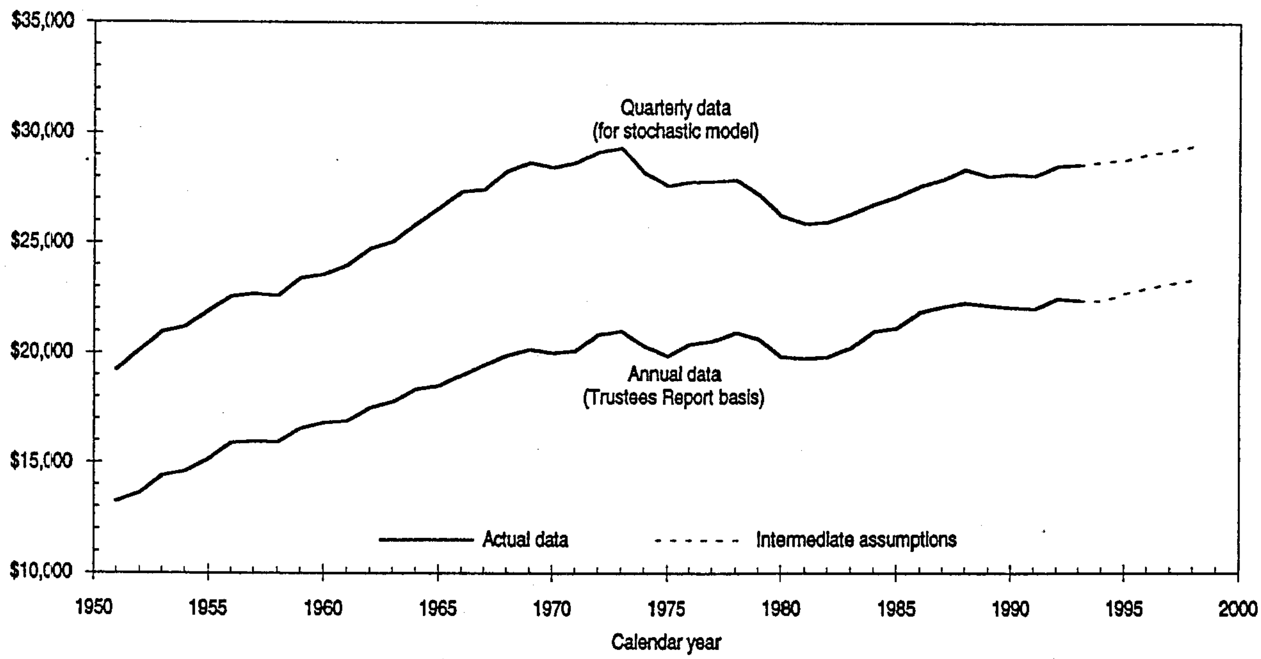
[47] The real wage differential is defined in the Trustees Report as the annual percentage increase in the average wages of employees covered under the Social Security program, minus the corresponding increase in the Consumer Price Index.

[48] The data on aggregate wage and salary accruals was adjusted to offset the shifting of an estimated $20 billion in wages from the first quarter of 1993 to the fourth quarter of 1992 and a similar amount from the first quarter of 1994 to the fourth quarter of 1993. These shifts are believed to have resulted from efforts by high-income wage earners to shelter a portion of their wages from the higher marginal income tax rates that became effective for 1993 and 1994 incomes.

For purposes of comparing forecasts from the statistical model (based on the quarterly data) with the Trustees Report assumptions, it was also necessary to derive quarterly values for future real wage increases on a basis consistent with the assumed real wage differentials shown in the Trustees Report. These quarterly "assumptions" were calculated based on the Trustees Report estimates for aggregate wages and salaries, by quarter, and on the report's estimates of the annual number of wage and salary workers.[49]

This procedure ensured that the evaluation of the assumptions for the future, based on a statistical analysis of the past data, would be consistent. The results might not be directly applicable, however, to the annual assumptions used in the Trustees Report. To help test the consistency of the procedure, chart E1 compares the series of real average wages based on the quarterly data to the corresponding annual amounts underlying the real wage differentials in the Trustees Report.

Appendix chart E1—Annual average wage, in constant 1993 dollars, as estimated from quarterly versus annual data



The historical data shown in chart E1 indicate that the real average wage based on the quarterly data was significantly higher during 1951-93 than the annual real average wage underlying the real wage differential. This difference in levels occurs primarily because of different definitions for the number of workers to be used in calculating the average wage. The real wage differential in the Trustees Report is based on the average wage among all persons having at least some wages at any time during the year. The *average* number of wage earners during the year is significantly

[49] For this purpose, quarterly numbers of wage and salary workers were estimated by the Revenue Projection Section of the Office of Research and Statistics, Social Security Administration, based on the annual employment figures from the Trustees Report.

lower—and it is this latter figure that is used, in effect, in defining the real average wage based on the quarterly data. Another difference between the two series is that the quarterly data are based on all employment and earnings, not just workers in jobs covered by the Social Security program. The effect of this difference, however, is relatively small.

The difference in the level of the average wage between the two series is relatively unimportant; the critical factor is the rate of increase in average wages. As indicated in chart E1, the year-by-year patterns of growth for the two series are very similar. The average annual rate of growth in real wages from 1951 to 1993 was 1.3 percent, based on the Trustees Report series, and 1.0 percent based on the quarterly data developed for this study.

Also shown in chart E1 are the assumed future values for the real average wage in 1994 through 1998, based on the quarterly values described above and the corresponding annual values from the Trustees Report. In both cases, the values shown are based on the intermediate assumptions. Once again, the average rates of increase are fairly similar, at 0.9 percent per year for the Trustees Report assumptions and 0.6 percent per year based on the quarterly assumptions derived from the Trustees Report estimates. The differential of 0.3 percent per year is the same as with the historical data.

Thus, both sets of quarterly values—that is, the actual past values and the assumed future values—appear to be reasonably consistent with the corresponding annual amounts from the Trustees Report. Consequently, the comparison of stochastic forecasts based on the quarterly historical data to the derived quarterly assumptions from the Trustees Report is believed to represent an appropriate substitute for a direct comparison based on the annual data underlying the Trustees Report assumptions.

The percentage increases in the real average wage from one quarter to the next are shown in the top panel of chart E2. In contrast to the other three economic time series studied in this report, the quarterly real wage increases appeared to exhibit relatively little autocorrelation. Values in one quarter seem little related to values in immediately preceding quarters. The search for outliers and level shifts turned up only two that were statistically significant—an additive outlier in 1952:IV and a level shift in 1973:II. Superimposed on the data is a plot of the average real wage increase during 1950:I through 1973:I and the corresponding average based on data in 1973:II and later. For each section of the graph, the real wage increase appears to vary about the applicable mean in a nearly random fashion.

The second and third panels in chart E2 present the autocorrelation and partial autocorrelation functions, respectively, for the real wage increase data after adjustment to remove the effect of the outlier and level shift described above. The ACF and PACF were little affected by removal of the additive outlier but were significantly affected by the adjustment for the level shift. Based on the original data, the ACF showed somewhat larger correlations at all lags (although only the first three exceeded two standard deviations). The partial autocorrelation coefficients were significantly larger at the first 4 lags. In both cases, the apparent additional correlation was spurious, representing only a by-product of the shift in the level of the data.

The level of autocorrelation present in the adjusted data was substantially lower than seen previously for the inflation rate, unemployment rate, and real interest rate. In particular, the ACF and PACF indicated only a modest degree of "persistence" in the data—the quarterly increases in the real wage data were much closer to being random and independent of prior increases. Whether this result is due to noise in the underlying data sources or to the combined effects of the various factors affecting wage growth is not clear.[50]

Several models were estimated based on the real wage data, with and without adjustment for the outlier and level shift. An AR(4) specification was indicated by the PACF in all cases, based on the significance of the coefficients at lags 1 and 4, but the parameter estimates were overstated somewhat in the absence of an adjustment for the level shift. In addition, the presence of the one-time level shift had significant implications for the probable accuracy of near-term forecasts (Chen and Tiao [28]). Accordingly, the final model incorporated an indicator variable to reflect the shift in 1973:II. The additive outlier in 1952:IV was also incorporated, primarily because its absence would have biased the parameter estimates downward even further below the already low level associated with the data as adjusted to remove the level shift.

Inclusion of the outlier and level shift should improve the accuracy of the stochastic model's near-term forecasts. At the same time, there is danger of understating the uncertainty associated with the forecasts because there could be additional outliers and/or level shifts in the future. This risk is minimized, however, by the very infrequent occurrence of such events in the past (i.e., only two statistically significant events in 43 years).

The resulting AR(4) intervention model is shown below. The t-statistics for the model parameters are shown in parentheses under the parameter estimates.

$$\begin{aligned} rw_t = \underset{(4.10)}{0.3534} + \underset{(2.98)}{0.2138}\, rw_{t-1} + \underset{(2.00)}{0.1437}\, rw_{t-2} + \underset{(1.32)}{0.0954}\, rw_{t-3} - \underset{(-2.93)}{0.2017}\, rw_{t-4} \\ + \underset{(3.61)}{2.3604}\, AO_{52:IV} - \underset{(-3.57)}{0.4011}\, LS_{73:II} + \varepsilon_t \end{aligned}$$

$$\text{where } AO_{52:IV} = \begin{cases} 0 \text{ for } t \neq 1952\text{:IV} \\ 1 \text{ for } t = 1952\text{:IV} \end{cases} \quad \text{and} \quad LS_{73:II} = \begin{cases} 0 \text{ for } t < 1973\text{:II} \\ 1 \text{ for } t \geq 1973\text{:II} \end{cases}$$

This model relates the current real wage increase (rw_t) to a weighted average of the increases in the prior 4 quarters (rw_{t-1} through rw_{t-4}).[51] The additive outlier is incorporated through the indicator variable $AO_{52:IV}$, which takes the value 1 in 1952:IV and 0 elsewhere. Its effect is to augment the estimated real wage increase by about 2.4 percentage points for that one quarter (and by rela-

[50] By way of comparison, the ACF and PACF of the annual real wage differential used in the Trustees Report also indicate minimal autocorrelation.

[51] The parameter estimate for rw_{t-3} was not statistically significant at the 5-percent level, but was left in the specification for comparison to other research results (e.g., Balke and Fomby [16]).

tively small amounts for another few quarters thereafter). The level shift is similarly incorporated through use of the variable $LS_{73:II}$.[52] Prior to 1973:II, the model estimates the real wage increase relative to a fixed mean of 0.47 percent per quarter (equivalent to about 1.9 percent per year). In 1973:II, the model begins a transition to lower real wage increases, leveling off at –0.06 percent per quarter (–0.24 percent per year) after a few quarters. The nature of this transition will be more apparent below, in the discussion of the model's forecasting performance within the sample period.

The bottom panel in chart E2 compares the actual quarterly real wage increases to the estimated values based on the statistical model. The standard error of the estimate (0.64 percent) represents only a modest improvement from the standard deviation of the original data (0.83 percent) and much of this reduction is attributable to incorporating the additive outlier and level shift into the model. The R^2 value, adjusted for degrees of freedom, is 42.4 percent. As shown in the chart, the estimated real wage increases reflect much less quarterly variation than exhibited by the actual data. This result occurs because, as noted above, the data show relatively little autocorrelation. Consequently, knowledge of recent experience is less useful in forecasting future real wage increases than was the case for the inflation rate, unemployment rate, and real interest rate.

Chart E3 presents the results of the various tests for model adequacy. The upper panel indicates that the standardized residuals of the model are random and independent, with only a few minor test failures (as often occur even with completely random data). The second panel shows a very high correlation between the residuals and a corresponding set of values drawn from a true normal distribution, indicating that the residuals are normally distributed. The plot of the residuals versus the fitted values from the model (third panel) shows no evidence of nonrandom patterns or changes in variance. The plot appears somewhat unusual because of the single large estimated value, associated with the additive outlier in 1952:IV. As noted above, all other estimated values tend to fall close to the applicable mean of the data since the autoregressive relationship to prior data values is weak.

The fourth panel of chart E3 indicates that the residuals show no significant sign of autocorrelation. The tests for nonconstant variance summarized in the final panel show no evidence of a problem. Thus, there was no need to employ an auxiliary GARCH model for estimating the variance of the error term through time. Overall, the model does not fit the data especially closely but the residuals meet all of the tests for model adequacy.

The model's suitability for forecasting is evaluated in chart E4. As with the other series studied in this report, the statistical model is used to generate 1-, 4-, and 20-step-ahead forecasts within the historical period for which actual data is available. For each such forecast, a 90-percent prediction interval is calculated and shown in gray. The 90-percent intervals should fail to capture the actual wage increase about 10 percent of the time. As indicated, this "outlier percentage" varies from 11.0 percent for the 1-step-ahead forecast intervals to 9.8 percent for the 20-step-ahead intervals—a result that is very close to expectations. Within the sample period, the statistical model appears

[52] The specification for this model, as shown above, differs somewhat from the traditional form of intervention models [22]. Due to the limitations of available computer software, the additive outlier was modeled as an *innovational* outlier and the level shift was modeled as a *gradual* level shift. The traditional and simplified models produce very similar results when the sum of the autoregressive coefficients is relatively small, as is the case for the real wage model.

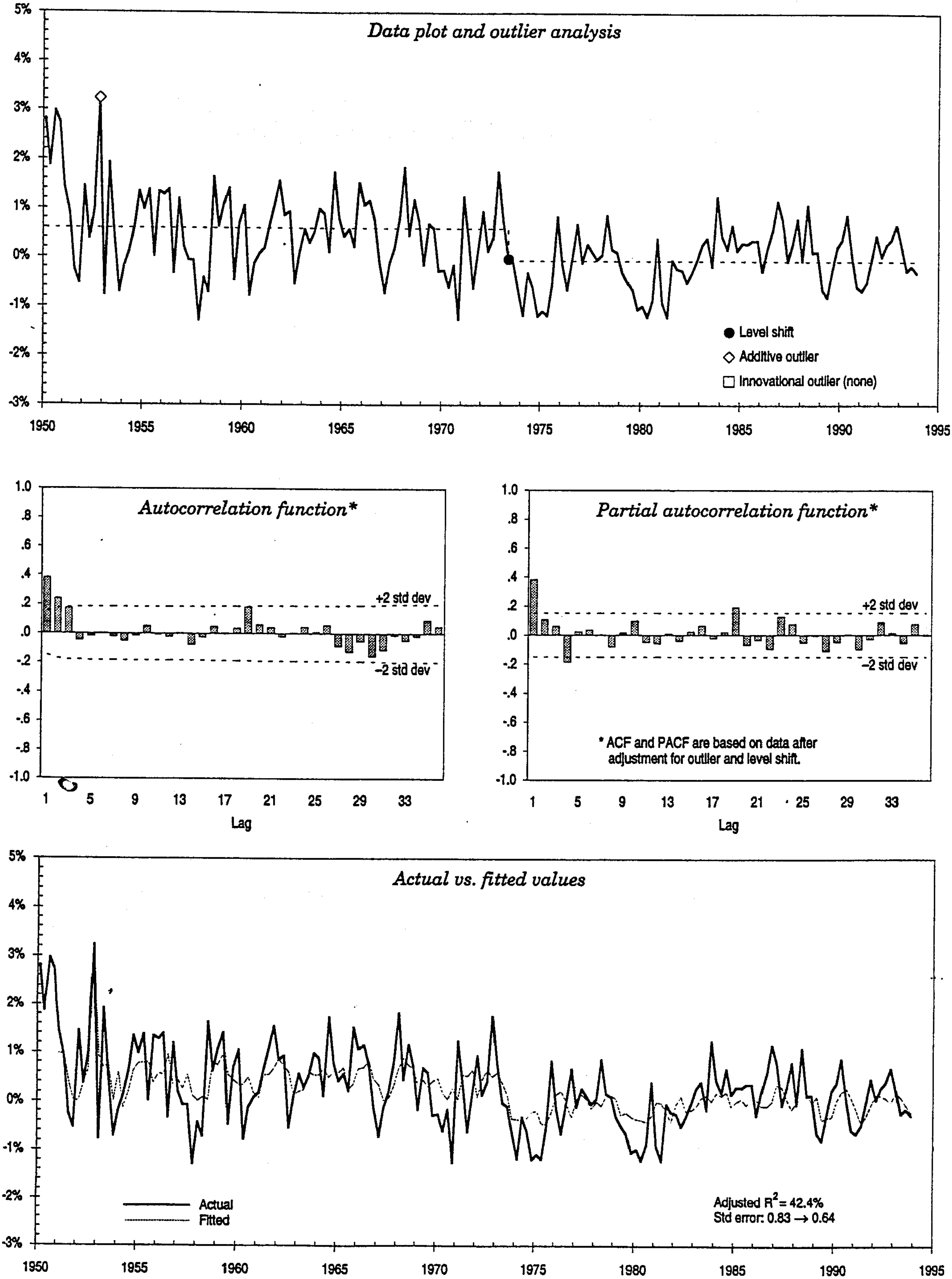
to perform consistently at forecast horizons of up to 20 quarters. As noted previously, these tests are performed assuming knowledge of the final model, including the level shift variable. In practice, the existence of the level shift would not be known until it had occurred and had been recognized in the data.

In contrast to the models described in appendices B, C, and D, the forecast intervals do not widen significantly for longer forecast horizons. The limited autocorrelation exhibited by the real wage data implies that knowledge of recent increases is relatively less useful in forecasting future increases than is the case for other series with greater autocorrelation. As a result, the predicted range of future variation is very similar to the actual past range even at short forecast horizons.

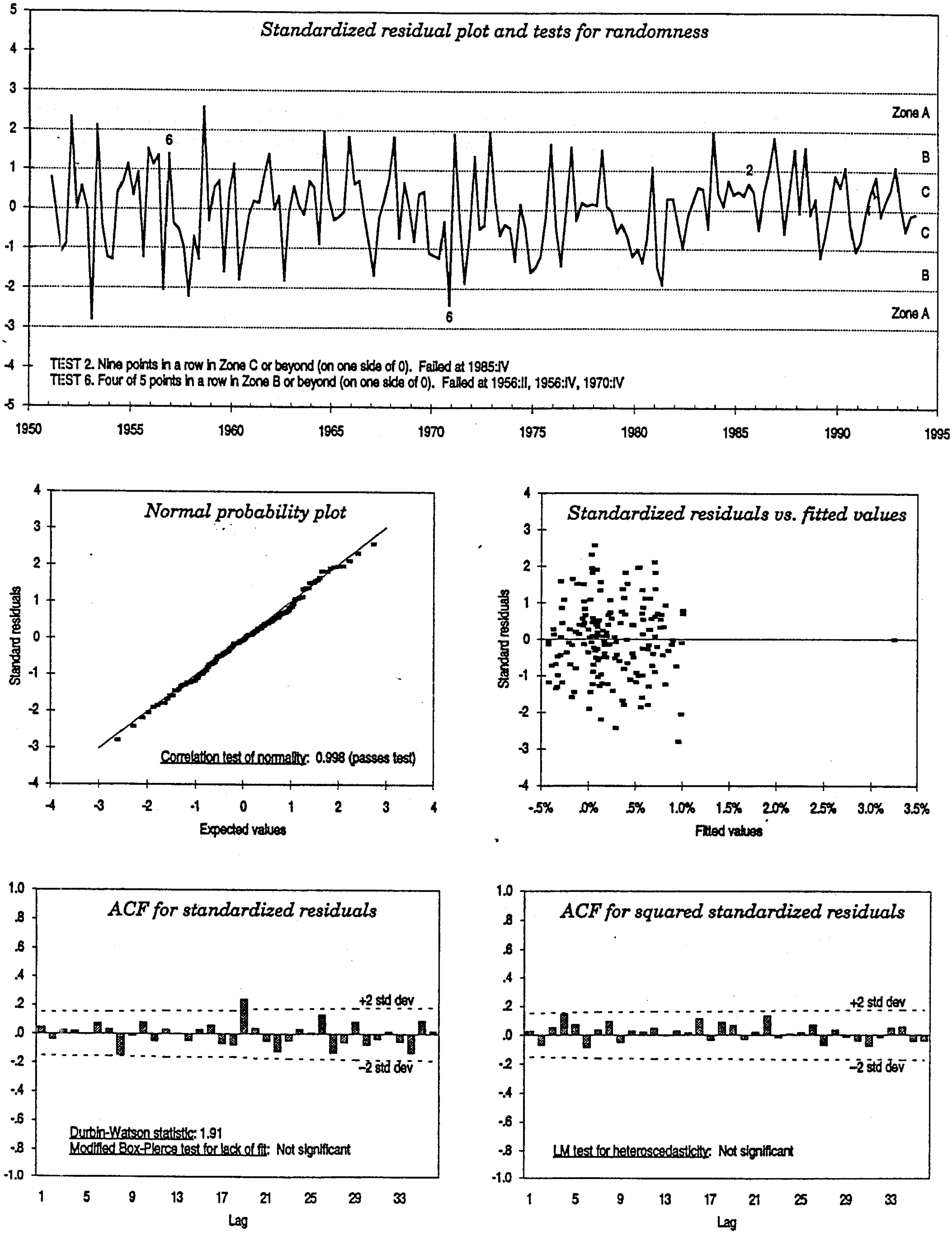
The forecast intervals are reduced somewhat by recognition of the level shift in 1973:II. As seen in each of the three panels in chart E3, the prediction intervals closely surround the actual wage increases before and after the level shift. Neither set of prediction intervals, however, would contain 90 percent of the actual data from the *opposite* time period.

The results of these various tests of data consistency, model specification, goodness-of-fit, and forecasting consistency suggest that the AR(4) intervention model for the real wage increase is appropriate for the intended use of evaluating the short-range real wage assumptions in the 1994 Trustees Report.

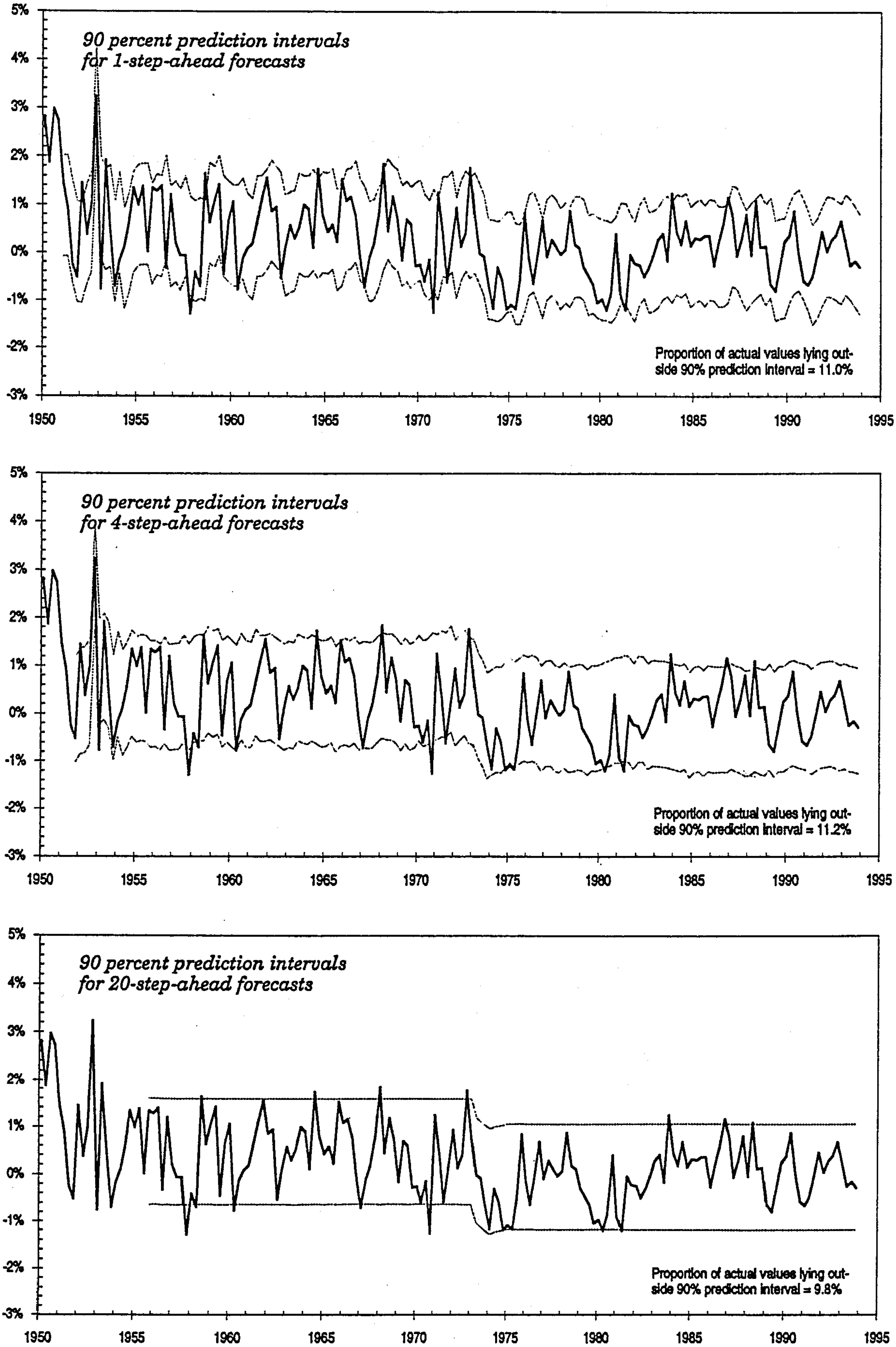
Appendix Chart E2—Identification and fit for real average wage increase model



Appendix Chart E3—Residual analysis for real average wage increase model



Appendix Chart E4—Within-sample forecast intervals for real average wage increase model



Appendix F—Data Underlying Stochastic Models

Table F1—Quarterly inflation rates, 1949:I through 1993:IV

(In percent)

Calendar year and quarter	Inflation rate	Calendar year and quarter	Inflation rate	Calendar year and quarter	Inflation rate	Calendar year and quarter	Inflation rate
1949:I	-1.9284	1961:I	0.1120	1973:I	1.3396	1985:I	0.6412
II	.2809	II	-.1119	II	2.2551	II	1.2424
III	-.2801	III	.4479	III	2.7376	III	.5664
IV	.0000	IV	.3344	IV	1.8505	IV	.9387
1950:I	-1.2640	1962:I	.1111	1974:I	2.8343	1986:I	.1860
II	.9957	II	.4440	II	2.8269	II	-.5569
III	2.3944	III	.2210	III	3.0241	III	.6223
IV	1.7882	IV	.5513	IV	2.9353	IV	.6184
1951:I	4.0541	1963:I	.0000	1975:I	1.8795	1987:I	1.1063
II	.9091	II	.1096	II	1.3359	II	1.3374
III	.0000	III	.8762	III	2.1971	III	1.1998
IV	1.6731	IV	.3257	IV	1.7199	IV	.8892
1952:I	-.2532	1964:I	.1082	1976:I	.9058	1988:I	.3788
II	.5076	II	.2162	II	1.2567	II	1.3078
III	1.1364	III	.3236	III	1.5957	III	1.2909
IV	.0000	IV	.5376	IV	1.1053	IV	1.1045
1953:I	-.7491	1965:I	.2139	1977:I	1.8987	1989:I	1.0084
II	.5031	II	.6403	II	1.9763	II	1.9135
III	.8761	III	.3181	III	1.4950	III	.5714
IV	.0000	IV	.5285	IV	1.1457	IV	.9740
1954:I	.0000	1966:I	.9464	1978:I	1.6181	1990:I	1.6077
II	.0000	II	.8333	II	2.6008	II	.8703
III	.0000	III	.8264	III	2.2763	III	1.8824
IV	-.3722	IV	1.0246	IV	2.0738	IV	1.7650
1955:I	-.2491	1967:I	.1014	1979:I	2.6264	1991:I	.4571
II	.0000	II	.7092	II	3.4766	II	.7553
III	.1248	III	1.1066	III	3.3598	III	.6051
IV	.4988	IV	.7960	IV	2.7540	IV	.8978
1956:I	-.3722	1968:I	.9872	1980:I	3.9104	1992:I	.4284
II	.7472	II	1.1730	II	3.6364	II	.8798
III	1.2361	III	1.2560	III	1.8360	III	.8721
IV	.8547	IV	1.2405	IV	2.7244	IV	.7205
1957:I	.7264	1969:I	1.0368	1981:I	2.7691	1993:I	.6438
II	.7212	II	1.6791	II	2.1252	II	.8529
III	1.1933	III	1.5596	III	2.7499	III	.3524
IV	.4717	IV	1.3550	IV	1.4105	IV	.7022
1958:I	.7042	1970:I	1.5152	1982:I	.8916		
II	.9324	II	1.6681	II	1.2725		
III	.1155	III	1.0363	III	2.0593		
IV	.1153	IV	1.3675	IV	.2736		
1959:I	-.1152	1971:I	.6745	1983:I	-.3070		
II	.2307	II	1.1725	II	1.3685		
III	.6904	III	1.1589	III	1.0800		
IV	.6857	IV	.3273	IV	.6344		
1960:I	.0000	1972:I	.9788	1984:I	.6304		
II	.4540	II	.7270	II	.6924		
III	.2260	III	.8019	III	1.6045		
IV	.6764	IV	.9547	IV	.5156		

Note: See appendix B for definition and source of data.

Table F2—Quarterly unemployment rates, 1950:I through 1993:IV

(In percent)

Calendar year and quarter	Unemploy-ment rate	Calendar year and quarter	Unemploy-ment rate	Calendar year and quarter	Unemploy-ment rate	Calendar year and quarter	Unemploy-ment rate
1949:I	—	1961:I	6.9	1973:I	4.9	1985:I	7.2
II	—	II	6.9	II	4.9	II	7.4
III	—	III	6.7	III	4.8	III	7.1
IV	—	IV	6.0	IV	4.9	IV	7.0
1950:I	6.3	1962:I	5.6	1974:I	5.1	1986:I	7.2
II	5.4	II	5.5	II	5.4	II	7.2
III	4.4	III	5.6	III	5.9	III	7.0
IV	4.3	IV	5.5	IV	7.2	IV	6.6
1951:I	3.4	1963:I	5.7	1975:I	8.6	1987:I	6.6
II	3.2	II	5.6	II	8.8	II	6.2
III	3.3	III	5.5	III	8.4	III	5.9
IV	3.1	IV	5.5	IV	8.2	IV	5.7
1952:I	2.9	1964:I	5.4	1976:I	7.6	1988:I	5.7
II	3.0	II	5.2	II	7.6	II	5.4
III	3.1	III	5.1	III	7.6	III	5.4
IV	2.7	IV	5.0	IV	7.8	IV	5.3
1953:I	2.6	1965:I	4.7	1977:I	7.4	1989:I	5.0
II	2.5	II	4.6	II	7.2	II	5.3
III	2.9	III	4.3	III	6.8	III	5.3
IV	4.5	IV	4.0	IV	6.4	IV	5.3
1954:I	5.7	1966:I	3.8	1978:I	6.3	1990:I	5.2
II	5.6	II	3.8	II	5.9	II	5.2
III	6.1	III	3.7	III	6.0	III	5.7
IV	5.0	IV	3.8	IV	6.0	IV	6.1
1955:I	4.6	1967:I	3.8	1979:I	5.8	1991:I	6.7
II	4.2	II	3.9	II	5.7	II	6.9
III	4.1	III	3.8	III	5.9	III	6.8
IV	4.2	IV	3.8	IV	6.0	IV	7.1
1956:I	4.2	1968:I	3.7	1980:I	6.3	1992:I	7.3
II	4.3	II	3.7	II	7.6	II	7.8
III	3.9	III	3.4	III	7.5	III	7.5
IV	4.2	IV	3.4	IV	7.2	IV	7.3
1957:I	3.7	1969:I	3.4	1981:I	7.4	1993:I	7.0
II	4.3	II	3.5	II	7.5	II	6.9
III	4.4	III	3.7	III	7.6	III	6.7
IV	5.2	IV	3.5	IV	8.5	IV	6.4
1958:I	6.7	1970:I	4.4	1982:I	9.0		
II	7.3	II	4.9	II	9.6		
III	7.1	III	5.4	III	10.1		
IV	6.2	IV	6.1	IV	10.8		
1959:I	5.6	1971:I	6.0	1983:I	10.3		
II	5.0	II	5.9	II	10.1		
III	5.5	III	6.0	III	9.2		
IV	5.3	IV	6.0	IV	8.3		
1960:I	5.4	1972:I	5.8	1984:I	7.8		
II	5.4	II	5.7	II	7.2		
III	5.5	III	5.5	III	7.3		
IV	6.6	IV	5.2	IV	7.3		

Note: See appendix C for definition and source of data.

Table F3—Quarterly real interest rates, 1961:I through 1993:IV

(Annualized rates, in percent)

Calendar year and quarter	Real interest rate	Calendar year and quarter	Real interest rate	Calendar year and quarter	Real interest rate	Calendar year and quarter	Real interest rate
1949:I	—	1961:I	2.2413	1973:I	2.4045	1985:I	7.3725
II	—	II	2.6967	II	.9928	II	7.0293
III	—	III	2.7170	III	.0204	III	7.4059
IV	—	IV	3.0671	IV	−1.6663	IV	6.4724
1950:I	—	1962:I	3.1920	1974:I	−2.9820	1986:I	6.1064
II	—	II	2.3743	II	−2.6177	II	6.4195
III	—	III	2.8534	III	−2.6631	III	6.4839
IV	—	IV	2.3849	IV	−4.0233	IV	6.6989
1951:I	—	1963:I	2.4986	1975:I	−3.5739	1987:I	5.4821
II	—	II	2.9648	II	−1.6984	II	4.4743
III	—	III	2.2960	III	−1.0181	III	4.2376
IV	—	IV	2.7727	IV	.0481	IV	4.1968
1952:I	—	1964:I	2.6616	1976:I	.8959	1988:I	4.2299
II	—	II	2.5525	II	.8570	II	4.8623
III	—	III	3.1173	III	1.5724	III	5.0086
IV	—	IV	2.9000	IV	1.8326	IV	4.3048
1953:I	—	1965:I	2.7915	1977:I	1.0762	1989:I	4.0125
II	—	II	2.3583	II	.3630	II	3.5129
III	—	III	2.3639	III	.4626	III	3.0592
IV	—	IV	2.6190	IV	.6568	IV	3.3119
1954:I	—	1966:I	2.2404	1978:I	1.2873	1990:I	3.1780
II	—	II	2.0447	II	.9043	II	4.8456
III	—	III	1.8937	III	.4812	III	2.7852
IV	—	IV	1.2728	IV	.0269	IV	2.2216
1955:I	—	1967:I	1.6413	1979:I	−1.0695	1991:I	2.9165
II	—	II	2.0101	II	−1.6813	II	3.0339
III	—	III	1.9701	III	−2.9351	III	4.4626
IV	—	IV	2.8097	IV	−2.0259	IV	4.6305
1956:I	—	1968:I	1.6668	1980:I	−3.0177	1992:I	4.4170
II	—	II	1.4408	II	−3.4951	II	4.6528
III	—	III	.9315	III	−2.2729	III	3.5272
IV	—	IV	.7271	IV	−.5803	IV	3.8309
1957:I	—	1969:I	1.2741	1981:I	.6359	1993:I	3.2452
II	—	II	.7701	II	3.3774	II	2.9091
III	—	III	.9422	III	2.9092	III	3.1984
IV	—	IV	1.1827	IV	4.4711	IV	2.9733
1958:I	—	1970:I	1.5294	1982:I	5.9495		
II	—	II	1.3051	II	6.3720		
III	—	III	1.5933	III	7.0917		
IV	—	IV	1.2260	IV	5.7955		
1959:I	—	1971:I	.9980	1983:I	7.3094		
II	—	II	1.6126	II	6.6035		
III	—	III	2.2074	III	9.1008		
IV	—	IV	2.1788	IV	8.5881		
1960:I	—	1972:I	2.1121	1984:I	7.4561		
II	—	II	2.5638	II	9.2691		
III	—	III	2.9270	III	8.9464		
IV	—	IV	2.5290	IV	7.8672		

Note: See appendix D for definition and source of data.

Table F4—Quarterly real wage increases, 1950:I through 1993:IV

(In percent)

Calendar year and quarter	Real wage increase	Calendar year and quarter	Real wage increase	Calendar year and quarter	Real wage increase	Calendar year and quarter	Real wage increase
1949:I	—	1961:I	.1805	1973:I	.7730	1985:I	.3002
II	—	II	.6411	II	–.0274	II	.2823
III	—	III	1.0976	III	–.0979	III	.3528
IV	—	IV	1.5552	IV	–.6453	IV	.3547
1950:I	2.8065	1962:I	.8563	1974:I	–1.1800	1986:I	–.2740
II	1.8682	II	.9386	II	–.3133	II	.2021
III	2.9641	III	–.5353	III	–.6029	III	.5767
IV	2.7239	IV	.0966	IV	–1.1982	IV	1.1586
1951:I	1.4454	1963:I	.5701	1975:I	–1.0914	1987:I	.7921
II	.9019	II	.2898	II	–1.1886	II	–.0721
III	–.2570	III	.5183	III	–.5444	III	.2944
IV	–.5327	IV	.9891	IV	.8403	IV	.8003
1952:I	1.4425	1964:I	.8926	1976:I	–.1464	1988:I	–.0510
II	.3680	II	.1123	II	–.6529	II	1.0911
III	.9967	III	1.7340	III	.0155	III	.1153
IV	3.2245	IV	.7877	IV	.6801	IV	.1338
1953:I	–.7643	1965:I	.4230	1977:I	–.1005	1989:I	–.6474
II	1.9203	II	.5735	II	.2723	II	–.8029
III	.5082	III	.2185	III	.1141	III	–.2845
IV	–.7087	IV	1.5219	IV	–.0424	IV	.2300
1954:I	–.1773	1966:I	1.0745	1978:I	.0665	1990:I	.3756
II	.1202	II	1.1582	II	.8728	II	.8781
III	.6094	III	.7614	III	.1712	III	.0103
IV	1.3342	IV	–.0996	IV	.0997	IV	–.5867
1955:I	.9850	1967:I	–.7227	1979:I	–.3032	1991:I	–.6686
II	1.3753	II	–.1213	II	–.4901	II	–.4901
III	.0175	III	.1828	III	–.6602	III	–.0277
IV	1.3344	IV	.8023	IV	–1.0539	IV	.4570
1956:I	1.2761	1968:I	1.8319	1980:I	–.9783	1992:I	.0319
II	1.3874	II	.4417	II	–1.2142	II	.2715
III	–.3320	III	1.1724	III	–.8713	III	.3801
IV	1.1986	IV	.7304	IV	.3891	IV	.6730
1957:I	.1970	1969:I	–.1605	1981:I	–.9144	1993:I	.2202
II	–.0722	II	.6874	II	–1.2166	II	–.2549
III	–.0746	III	.5889	III	–.0519	III	–.1693
IV	–1.2854	IV	–.2837	IV	–.2261	IV	–.2968
1958:I	–.4189	1970:I	–.2617	1982:I	–.2630		
II	–.7097	II	–.6072	II	–.5083		
III	1.6334	III	–.1575	III	–.3083		
IV	.6272	IV	–1.2600	IV	–.0380		
1959:I	1.0793	1971:I	1.2380	1983:I	.2659		
II	1.4090	II	.3877	II	.4015		
III	–.4605	III	–.6253	III	–.1738		
IV	.6834	IV	.2052	IV	1.2371		
1960:I	1.0559	1972:I	.9365	1984:I	.4292		
II	–.7732	II	.1303	II	.1739		
III	–.1271	III	.4101	III	.6675		
IV	.0796	IV	1.7530	IV	.1483		

Note: See appendix E for definition and source of data.

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